

IONOSPHERIC TOMOGRAPHY USING FARADAY ROTATION OF  
AUTOMATIC DEPENDENT SURVEILLANCE BROADCAST (UHF)  
SIGNALS

Ionospheric Measurement From ADS-B Signals

LA TOMOGRAPHIE IONOSPHERIQUE EN UTILISANT LA ROTATION  
DE FARADAY DES SIGNAUX ADS-B (UHF)

Mesure ionosphériques Des Signaux l'ADS-B

A Thesis Submitted

to the Division of Graduate Studies of the Royal Military College of Canada

by

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*This work is dedicated to my loving wife Caroline, and my parents Mark and Donna, who have unselfishly sacrificed much to support me in all my endeavours. Their moral support during challenging times has kept me on track.*

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## Abstract

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The proposed launch of a CubeSat carrying the first space-borne ADS-B receiver by RMCC will create a unique opportunity to study the modification of radio waves following propagation through the ionosphere as the signals propagate from the transmitting aircraft to the passive satellite receiver(s). Experimental work is described which successfully demonstrated that ADS-B data can be used to reconstruct two-dimensional electron density maps of the ionosphere using techniques from computerized tomography. Ray-tracing techniques are used to determine the characteristics of individual waves, including the wave path and the state of polarization at the satellite receiver. The modelled Faraday rotation is determined and converted to TEC along the ray-paths. The resulting TEC is used as input for CIT using ART. This study concentrated on meso-scale structures 100–1000 km in horizontal extent. The primary scientific interest of this thesis was to show the feasibility of a new method to image the ionosphere and obtain a better understanding of magneto-ionic wave propagation.

**Keywords:** Automatic Dependent Surveillance-Broadcast (ADS-B), Faraday rotation, electromagnetic (EM) waves, radio frequency (RF) propagation, ionosphere (auroral, irregularities, instruments and techniques), electron density profile, total electron content (TEC), computer ionospheric tomography (CIT), algebraic reconstruction technique (ART)

## Résumé

Cushley, Alex Clay. M.Sc. Collège militaire royal du Canada, 30 mai, 2013. *LA TOMOGRAPHIE IONOSPHERIQUE EN UTILISANT LA ROTATION DE FARADAY DES SIGNAUX ADS-B (UHF): Mesure ionosphériques Des Signaux l'ADS-B.*  
Thèse dirigée par Dr. Jean-Marc Noël, Ph.D.

Le projet de lancement d'un nanosatellite de type "CubeSat" avec un récepteur ADS-B par le CMRC constituera une occasion unique d'étudier l'effet de l'ionosphère sur les signaux transmis par un aéronef et reçus passivement par un satellite. Des travaux expérimentaux sont décris dans lesquels on démontre que les données d'ADS-B peuvent être utilisées pour reconstruire des cartes de densité électrons bidimensionnels de l'ionosphère à l'aide des techniques de scanographie. Des techniques de traçage de rayons sont utilisées pour déterminer les caractéristiques de l'onde, y compris le trajet d'onde et la polarisation complète, au récepteur du satellite. La rotation de Faraday est modélisé et convertie en contenu électronique total (TEC) le long des trajectoires des rayons. Le TEC est utilisé comme entrée pour la scanographie ionosphérique en utilisant une technique de reconstruction algébrique. Cette étude concentre sur les structures de taille moyenne entre 100 km et 1000 km dans la direction horizontale. L'intérêt scientifique principal de cette thèse est de démontrer la capacité d'une nouvelle technique pour produire des images de la densité électrique. De plus, on pourra obtenir une meilleure compréhension de la propagation des ondes magnétoioniques.

**Mots clés :** L'ADS-B, rotation de Faraday, ondes électromagnétiques, propagation radioélectrique, l'ionosphère (auroral, irrégularités, instruments et techniques), le profil de densité d'électrons, contenu électronique total, tomographie informatisée ionosphérique, technique de reconstruction algébrique)

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## List of Acronyms

2D	Two-dimensional
ADS-B	Automatic Dependent Surveillance Broadcast
AOI	Area of Interest
ART	Algebraic Reconstruction Technique
ASL	Above Sea Level
ATC	Air Traffic Control
ADCS	Attitude Determination and Control
CF	Canadian Forces
CIT	Computerized Ionospheric Tomography
CMEs	Coronal Mass Ejections
CT	Computerized Tomography
DART	Direct Algebraic Reconstruction Technique
<i>dTEC</i>	Differential Total Electron Content
DX	Distant Earth Communications
EM	Electromagnetic
ePOP	Enhanced Polar Outflow Probe Eqn.
Equation	
ES	Extended Squitter
FFT	Fast Fourier Transform
FL	Flight Level
FLOAT	Flying Laboratory for the Observation of ADS-B Transmissions
FOV	Field-of-view
FR	Faraday Rotation
ft	feet
GNSS	Global Navigation Satellite System
GPS	Global Positioning System
HF	High-frequency
IGRF	International Geomagnetic Reference Field
IRI	International Reference Ionosphere
ISR	Incoherent Scatter Radar
LHCP	Left-hand Circular Polarized
LAN	Local Area Network
LEO	Low Earth Orbit
LOS	Line of sight
MART	Multiplicative Reconstruction Technique
MEO	middle-Earth Orbit
Mode-S	Mode Select
Nav Canada	Navigation Canada
$n_e$	Electron Density
PCA	Polar Cap Absorption
QL	Quasi-longitudinal
QT	Quasi-transverse
RHCP	Right-hand Circular Polarized

RMCC	Royal Military College of Canada
SART	Simultaneous Algebraic Reconstruction Technique
SIRT	Simultaneous Iterative Reconstruction Technique
SMAD	Space Mission Analysis and Design
STEC	Slant TEC
STK	Satellite Tool Kit
TEC	Total Electron Content
TECu	Total Electron Content Units
UV	Ultraviolet
VDL	VHF Data Link
VHF	Very high frequency
UAT	Universal Access Transceiver
USA	United States of America

## Nomenclature

$\chi$	.....	The ellipticity angle (rad)
$\Delta\phi$	.....	The phase difference (rad)
$\Delta UT$	.....	The local time difference between transmitter and receiver (s)
$\Delta_j$	.....	The uncertainty associated with pixel $j$ (electrons·m $^{-2}$ )
$\epsilon_0$	.....	The permittivity of free space ( $8.85418782 \times 10^{-12} \text{C}^2/\text{N}\cdot\text{m}^2$ )
$\eta$	.....	The distance $y$ along the path in rotated coordinates (m)
$\lambda$	.....	The wavelength of the EM wave (m)
$\lambda_k$	.....	The relaxation parameter (dimensionless)
$\Omega$	.....	The Faraday rotation of an EM wave (rad)
$\omega_p$	.....	The angular plasma frequency (rad·s $^{-1}$ )
$\omega_p$	.....	The angular radio wave frequency (rad·s $^{-1}$ )
$\phi_n$	.....	The $n^{th}$ carrier phase (rad)
$\psi$	.....	Orientation angle (rad)
$\rho$	.....	The number of pixels in the reconstruction grid (dimensionless)
$\tau$	.....	The tolerance (electrons·m $^{-2}$ )
$\theta$	.....	Aspect angle (rad)
$\vec{B}$	.....	The magnetic field vector of the EM-wave (nT)
$\vec{E}$	.....	The electric field vector of the EM-wave (N·C $^{-1}$ )

$\vec{p}$	The vector of the measured, integrated quantity (electrons·m <sup>-2</sup> )
$\vec{x}$	The vector of unknown values (electrons·m <sup>-3</sup> )
$\xi$	The sensor position, relative to the centre of rotation (m)
$A$	The geometry matrix (m)
$a$	The semi-major axis (m)
$B$	The magnetic field strength (T)
$b$	The semi-minor axis (m)
$B_{avg}$	The average magnetic field (nT)
$b_z(l)$	The component of the magnetic field in the direction of the propagation at a point along path $l$ (T)
$c$	The speed of light in a vacuum ( $3 \times 10^8$ m·s <sup>-1</sup> )
$d$	The distance between transmitter and receiver (m)
$d_{ij}$	The contribution to pixel $j$ from ray $i$ in the logical matrix $D$ (m)
$D_i$	The $i^{th}$ row of $D$ (m)
$e$	The charge of an electron ( $1.60217657 \times 10^{-19}$ C)
$E_L$	The left-hand circular polarized electric vector of the wave (N·C <sup>-1</sup> )
$E_R$	The right-hand circular polarized electric vector of the wave (N·C <sup>-1</sup> )
$E_x$	The $x$ component of the electric vector of the wave (N·C <sup>-1</sup> )
$E_y$	The $y$ component of the electric vector of the wave (N·C <sup>-1</sup> )
$E_{i \times 1}$	The column vector of $i$ error values (electrons·m <sup>-2</sup> )
$F$	The Fourier transform of a function (dimensionless)

$f$	.....	The radio wave frequency (Hz)
$f(x, y)$	...The attenuation coefficient as a function of position in the reconstruction grid pixel $(x,y)$ ( $\text{dB}\cdot\text{m}^{-1}$ )	
$f_c$	.....	The gyrofrequency (Hz)
$f_n$	.....	The $n^{th}$ carrier frequency (Hz)
$f_{crit}$	.....	The critical Frequency (MHz)
$f_p$	.....	The plasma frequency (Hz)
$I$	.....	The received intensity (V)
$i$	.....	The imaginary component of the complex number( $i^2=-1$ dimensionless)
$I_0$	.....	The transmitted intensity (V)
$k$	.....	The iteration number (dimensionless)
$k_\eta$	.....	The co-ordinate $\eta$ in transform space (m)
$k_\xi$	.....	The co-ordinate $\xi$ in transform space (m)
$k_x$	.....	The attenuation function co-ordinate $x$ in transform space (m)
$k_y$	.....	The attenuation function co-ordinate $y$ in transform space (m)
$l$	.....	The path length (m)
$LT_{ADS-B}$	.....	The local time at each ADS-B data point (s)
$LT_{slice}$	.....	The local time at the slice of longitude (s)
$m$	.....	The mass of an electron ( $9.10938291 \times 10^{-31}$ kg)
$n$	.....	Index of refraction (dimensionless)
$N^{k+1}$	.....	The vector of the modified pixel values $n_j^{k+1}$ for the next iteration (electrons $\cdot$ m $^{-3}$ )

$N^k$	.....	The vector of pixels $n_j^k$ for the iteration (electrons·m <sup>-3</sup> )
$n_e$	.....	The electron density (electrons·cm <sup>-3</sup> )
$n_e(l)$	.....	The electron density at a point along path $l$ (electrons·m <sup>-3</sup> )
$n_0$	.....	The pixel value for the first iteration (electrons·m <sup>-3</sup> )
$N_e(x)$	.....	The electron density as a function of pixel $x$ (electrons·m <sup>-3</sup> )
$N_{j \times 1}$	.....	The column vector of $j$ unknown pixel values (electrons·m <sup>-3</sup> )
$n_j$	.....	The brightness of pixel $j$ (electrons·m <sup>-3</sup> )
$n_j^k$	.....	The brightness of pixel $j$ for the iteration $k$ (electrons·m <sup>-3</sup> )
$n_j^k$	.....	The brightness of pixel $j$ for the next iteration (electrons·m <sup>-3</sup> )
$n_x$	..	The number of pixels in the horizontal ( $x$ ) dimension of the reconstruction grid (dimensionless)
$n_y$	..	The number of pixels in the horizontal ( $y$ ) dimension of the reconstruction grid (dimensionless)
$p$	.....	The wave polarization ratio (dimensionless)
$p(\xi, \phi)$	.....	The projection intensity at sensor position $\xi$ , for rotation angle $\phi$ (V)
$q$	.....	The charge of a particle (C)
$R$	.....	The Radon trasform of function $f$ (dimensionless)
$r$	.....	The gyroradius of a particle (m)
$r_e$	.....	The radius of an electron ( $2.81794 \times 10^{-15}$ m)
$RM$	.....	The measured Faraday rotation between consecutive epochs (rad·m <sup>-2</sup> )
$STEC$	.....	The slant $TEC$ (as opposed to vertical $TEC$ ) (electrons·m <sup>-2</sup> )
$STEC_{i \times 1}$	.....	The column vector of $i$ measurements (electrons·m <sup>-2</sup> )

$STEC_i$	.....	The $STEC$ measurement for the $i^{th}$ ray-path ( $\text{electrons}\cdot\text{m}^{-2}$ )
$t$	.....	The time (s)
$TEC$	.....	The total electron content along the ray-path or plasma column ( $\text{m}^{-2}$ )
$v$	.....	The velocity of a particle ( $\text{m}\cdot\text{s}^{-1}$ )
$v_\phi$	.....	The phase velocity ( $\text{rad}\cdot\text{s}^{-1}$ )
$X$	..	Squared ratio of plasma frequency to radio wave frequency (dimensionless)
$Y$	.....	Ratio of gyrofrequency to radio wave frequency (dimensionless)
$D_{i\times j}$	.....	The geometry matrix (m)

