EXPEDITING SUBMARINE POSITIONING SOLUTION CONVERGENCE WITH THE INTEGRATION OF GALILEO

ACCÉLÉRATION DE LA CONVERGENCE DES SOLUTIONS DE POSITIONNEMENT DES SOUS-MARINS AVEC L'INTÉGRATION DE GALILEO

A Thesis Submitted to the Division of Graduate Studies
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by

Kevin Anthony Hunt, CD, MEng, MSc, rmc
Lieutenant-Commander

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Dedication

For the smart navigation of submarines and safety of their crews encapsulated within, for the journey beneath the waves is already perilous enough without the risk of navigation errors.

Acknowledgements

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All without which I wouldn’t be standing where I am today.

Abstract

For safe navigation, submarines rely on two complex systems - satellite navigation and inertial navigation – which are integrated to maximize technological capability. As complementary systems, the inertial navigation system requires periodic position updates from global navigation satellite systems to nullify the inherent positioning uncertainty that accumulates while the submarine is underwater and out of satellite connectivity. When submarines return to the surface, they aim to remain covert as much as possible, which includes limited antenna exposure above the surface of the water. Thus satellite positioning updates must be attained as quickly as possible.

The initial fix of any satellite navigation receiver is relatively inaccurate, with the solution improving as more satellites are acquired and information is processed. While solution convergence times can span many minutes to reach maximum accuracy, submarines only need to expose their antenna long enough to get a solution that meets the navigation system's accuracy threshold. A novel term "Time to First Good Fix" is introduced in this paper, quantifying the duration between antenna exposure and a first position estimate that meets the submarine's accuracy threshold. A receiver that maintains a low time to the first "good fix" ensures the submarine can dive again with updated navigation information while spending minimal time at or near the surface.

Previous studies on multi-GNSS receivers outline their benefits relative to single-GNSS receivers, including increased positioning accuracy and system reliability. This thesis investigates whether a GPS+Galileo receiver computes a position estimate that meets an accuracy threshold quicker than a GPS-only receiver, particularly in a remote maritime environment where it cannot benefit from the number of augmentation services available to GNSS users. A series of side-by-side, full field of view tests with a GPS-only receiver and a GPS+Galileo receiver are conducted to analyze the differences in solution convergence times. The results show that a GPS+Galileo receiver can attain a 4-meter positioning accuracy 27 seconds or 45% faster than a GPS-only receiver. Despite the GPS-only receiver always outputting a position estimate quicker, the GPS+Galileo receiver was always more accurate and provided accurate positioning data quicker regardless of the user's accuracy threshold. Although GPS and Galileo work with different message structures and data rates, their interoperability results in a GPS+Galileo receiver having more available satellites and more optimal satellite geometry which results in expedited solution convergence. The minutes following the first accurate positioning data are also more stable, enabling better positioning data input for the submarine's integrated navigation system for underwater navigation. Galileo's interoperability with GPS, worldwide availability, continuing operational maturity, and the availability of multi-GNSS receivers mean that submarines could enhance their navigation and operational capability with minimal changes to procedures, requirements, infrastructure, and signal processing. These results shows that a receiver that capitalizes on the similarities and availability of both GPS and Galileo could offer submarines expedited fixing times for similar cost and negligible change to their current navigation system configuration.

Keywords: Accuracy, Convergence, Galileo, GNSS, GPS, Navigation, Positioning, Submarine, Time to First Fix.

Résumé

Pour une navigation sûre, les sous-marins s'appuient sur deux systèmes complexes - la navigation par satellite et la navigation inertielle - qui sont intégrés pour maximiser la capacité technologique. En tant que systèmes complémentaires, le système de navigation inertielle nécessite des mises à jour périodiques de position à partir des systèmes mondiaux de navigation par satellite pour annuler l'incertitude de positionnement inhérente qui s'accumule lorsque le sous-marin est sous l'eau et hors de la connectivité satellite. Lorsque les sous-marins remontent à la surface, ils visent à rester aussi discrets que possible, ce qui inclut une exposition limitée de l'antenne au-dessus de la surface de l'eau. Ainsi, les mises à jour de positionnement par satellite doivent être obtenues aussi rapidement que possible.

La position initiale de tout récepteur de navigation par satellite est relativement imprécise, la solution s'améliorant à mesure que davantage de satellites sont acquis et que les informations sont traitées. Bien que les temps de convergence de la solution puissent s'étendre sur de nombreuses minutes pour atteindre une précision maximale, les sous-marins n'ont besoin d'exposer leur antenne que suffisamment longtemps pour obtenir une solution répondant au seuil de précision du système de navigation. Un terme novateur "Temps pour le premier bon positionnement" est introduit dans cet article, quantifiant la durée entre l'exposition de l'antenne et une première estimation de position qui répond au seuil de précision du sous-marin. Un récepteur qui maintient un faible temps pour le premier "bon positionnement" assure que le sous-marin peut plonger à nouveau avec des informations de navigation mises à jour tout en passant un temps minimal à ou près de la surface.

Les études précédentes sur les récepteurs multi-GNSS soulignent leurs avantages par rapport aux récepteurs mono-GNSS, notamment une précision de positionnement accrue et une fiabilité du système. Cette thèse examine si un récepteur GPS+Galileo calcule une estimation de position qui répond à un seuil de précision plus rapidement qu'un récepteur GPS seul, en particulier dans un environnement maritime éloigné où il ne peut pas bénéficier du nombre de services d'augmentation disponibles pour les utilisateurs de GNSS. Une série de tests côte à côte, avec un récepteur GPS seul et un récepteur GPS+Galileo, sont menés pour analyser les différences dans les temps de convergence de la solution. Les résultats montrent qu'un récepteur GPS+Galileo peut atteindre une précision de positionnement de 4 mètres 27 secondes ou 45% plus rapidement qu'un récepteur GPS seul. Bien que le récepteur GPS seul fournisse toujours une estimation de position plus rapidement, le récepteur GPS+Galileo était toujours plus précis et fournissait des données de positionnement précises plus rapidement, quel que soit le seuil de précision de l'utilisateur. Bien que le GPS et Galileo fonctionnent avec des structures de messages et des débits de données différents, leur interopérabilité résulte en un récepteur GPS+Galileo ayant plus de satellites disponibles et une géométrie satellitaire plus optimale, ce qui accélère la convergence de la solution. Les minutes suivant les premières données de positionnement précises sont également plus stables, permettant une meilleure entrée de données de positionnement pour le système de navigation intégré du sous-marin pour la navigation sous-marine. L'interopérabilité de Galileo avec le GPS, sa disponibilité mondiale, sa maturité opérationnelle continue et la disponibilité des récepteurs multi-GNSS signifient que les sous-marins pourraient améliorer leur capacité de navigation et opérationnelle avec des changements minimes aux procédures, exigences, infrastructure et traitement du signal. Ces résultats montrent qu'un récepteur qui tire parti des similitudes et de la disponibilité à la fois du GPS et de Galileo pourrait offrir aux sous-marins des temps de fixation accélérés pour un coût similaire et un changement négligeable de leur configuration actuelle du système de navigation.

Mots-clefs : convergence, exactitude, Galileo, GNSS, Navigation, Positionnement, Sous-marin, Système de Positionnement Global, Temps de première fixation.

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List of Acronyms

A-GPS	Assisted GPS
A-PNT	Assured Positioning, Navigation, and Timing
C/A	Coarse Acquisition
CEP	Circular Error Probable
DGPS	Differential Global Positioning System
DoD	Department of Defense
DOP	Dilution of Precision
DR	Dead-Reckoning
ECDIS	Electronic Chart Display and Information Systems
EP	Estimated Position
ESA	European Space Agency
EU	European Union
EUSPA	European Union Agency for the Space Programme
FCC	Federal Communications Commission
FDE	Fault Detection and Exclusion
FOV	Field of View
GBAS	Ground-Based Augmentation System
GDOP	Geometric Dilution of Precision
GEO	Geostationary
GLONASS	Global Navigation Satellite System
GNSS	Global Navigation Satellite System
GPS	Global Positioning System
HDOP	Horizontal Dilution of Precision
IMU	Inertial Measurement Unit
INS	Inertial Navigation System
LBS	Location-Based Services
LEO	Low Earth Orbit
LOS	Line-of-sight
MEO	Medium Earth Orbit
NATO	North Atlantic Treaty Organization
NAVSTAR	Navigation Satellite Timing and Ranging
NNSS	Navy Navigation Satellite System
NVM	Non-Volatile Memory
OOW	Officer of the Watch
PD	Periscope Depth
PDOP	Position Dilution of Precision
PNT	Positioning, Navigation, and Timing
POE	Pool of Errors
PPP	Precise Point Positioning
PPS	Precise Positioning Service
PRN	Pseudorandom noise

P/Y	Precision/Secure
RADAR	Radio Detection and Ranging
RAIM	Receiver Autonomous Integrity Monitoring
RCN	Royal Canadian Navy
R&D	Research and Development
RTK	Real-Time Kinematics
RTPPP	Real-Time Precise Point Positioning
SAR	Search and Rescue
SBAS	Space-Based Augmentation System
SHINNADS	Shipboard Integrated Navigation and Display System
SPS	Standard Positioning Service
TOF	Time of Flight
TTFF	Time to First Fix
TTFGF	Time to First Good Fix
U.S.	United States
UTC	Coordinated Universal Time
WADGPS	Wide Area Differential Global Positioning System
WIFI	Wireless Fidelity

Preface

This work outlines the use of satellite navigation and inertial navigation systems onboard submarines; however, all contents of this thesis are unclassified. All discussed details pertaining to submarines systems, capabilities, and navigation protocol are available in open literature.

Chapter 1

Introduction

It was not long ago that ships at sea estimated their position based on the Sun and the stars. The publication of ocean charts and advancements in mechanical time-keeping revolutionized how sailors kept track of their voyages and introduced dead-reckoning. With the launching of satellites and advancement of precision time-keeping through the twentieth century, satellite navigation quickly revolutionized how sailors could navigate anywhere on Earth, in any weather, day or night. Modern marine navigation relies heavily on satellite technology, rendering methods of traditional navigation such as magnetic compasses and sextants obsolete. The availability and reliability of high-accuracy satellite navigation has become so commonplace that it is arguably taken for granted by today's mariners until they are suddenly unavailable.

Onboard submarines, accurate positioning data plays a critical role in the functionality of sensor, communication, and weapon systems. If positioning information is not readily available, today's war fighters have a serious vulnerability. This section reviews the use of satellite positioning data onboard submarines in the Royal Canadian Navy before introducing the problem with re-acquiring satellite signals in the remote maritime environment. The research problem of abbreviating submarine fixing times which forms the basis of this thesis is then presented.

1.1 Submarine Navigation

Ships and submarines of the Royal Canadian Navy (RCN), as well as small boats, maritime patrol aircraft, and helicopters all rely on the Global Positioning System (GPS) for high-accuracy, real-time positioning information. Unlike GPS users inland, their operations in remote maritime environments mean that marine GPS receivers do not have access to the same augmentation services that offer reduced fixing times and enhanced positioning accuracy. On the other hand, the remoteness of open waters results in a full field of view for maximum satellite visibility and minimal interference and noise, offering the best possible conditions for satellite acquisition, satellite geometry, and data download.

The RCN operates four diesel-electric VICTORIA Class patrol submarines (Government of Canada, 2024). Each submarine has redundant GPS antennas and Precise Positioning Service (PPS) receivers. When GPS is available, it is the submarine's primary source of positioning and timing information as its services are all-weather, 24-hour, and worldwide. From the receivers, GPS positioning data is fed to the submarine's navigation data network, which distributes data to end-users including communication, sensor, fire control, weapon, and navigation systems (Northrop Grumman, 2023). However, the GPS information is not augmented by any terrestrial or aimed space-based augmentation services of which inland GNSS users have become accustomed to for faster and more accurate positioning data. As a result, submarines are obliged to rely exclusively on satellite signals, which are slow and subject them to various positioning errors.

RCN ship and submarine navigation is assisted by the Shipboard Navigation and Display System (SHINNADS) running an Electronic Chart Display and Information System (ECDIS) software for an electronic moving chart display. This system receives positioning data for the purpose of safe navigation. Redundant SHINNADS computers enable the Officer of the Watch (OOW) and Navigating Officer (NavO) to plan and execute voyage plans in all charted waterways. Laptops and tablets with built-in SPS receiver capability are available for voyage planning and navigation, but do not feed GPS information to the end-users. Navigators also possess commercially-available handheld GPS units for back-up positioning data.

GPS information is also sent to strap-down Inertial Navigation Systems (INS) which provide their own independent, self-contained, high-performance positioning solution to the submarine. With an initial position and record of all linear and rotational acceleration over time, an INS computes a continuous series of position, velocity, and attitude estimates without external input, thus eliminating risk of electromagnetic interference (jamming) from the outside world (Cole, 2015; Hess, 2015; Keller, 2008). The vehicle's change in position and velocity can be calculated by integrating all sensed accelerations with respect to time (Marvel, 1998):

$$x = \int v \, dt = \iint a \, dt \, dt \tag{1}$$

In a perfect world, an INS can provide a submarine with assured position navigation and timing (A-PNT), an indefinite estimated position (EP) considering its initial position and the sequence of subsequent movement in all three axes for continued navigation and military operations (Keller, 2023). Together, an integrated[1] GPS/INS overcomes the drawbacks of standalone INS and GPS receivers (Table 1) to provide the submarine with an accurate, long-term navigation solution. The complementary properties of GPS and INS have led to their integration onboard military vehicles, including submarines, ships, and aircraft, for an autonomous, reliable, and accurate navigation system (Boguspayev *et al*., 2023; Schmidt & Phillips, 2010). Together they provide the backbone of the

[1] The simultaneous use of two or more technologies (Hess, 2015).

submarine's stealthy navigation capability, providing reliable and accurate positioning data to weapons and sensors systems for their precise operation (Cahyadi, Asfihani, Madriyanto, & Erfianti, 2022).

Table 1. Relative comparisons of GNSS and INS, showing the advantages and disadvantages of both systems.

Parameter	GNSS	INS
Long-term Position Drift	Low	High
Data Rate	Low	High
Time to Initialize / Converge	Low	High
Susceptibility to Interference	High	Nil
Cost of User Infrastructure	Low	High
Size	Low	High

However, the accuracy of the submarine's INS EP degrades with time as a result of combining alignment, sensor, and computation errors. With GPS connectivity, positioning error can be nullified with the latest positioning data and the system designed to self-correct for known errors for better long-term accuracy. Without GPS position updates, even the smallest alignment error, sensor bias, and sensor errors lead to an accumulation of error in an inertial EP over time, while acceleration measurement inaccuracies fed through double integration for positioning combine with those errors that results in decreased INS accuracy over time, where total error is limitless (Boguspayev *et al*., 2023; Cahyadi, Asfihani, Madriyanto, & Erfianti, 2022; Goward, 2022; Schmidt, 2015). Although the design and manufacturing of inertial navigation systems are continuously being refined for capability and performance, a submarine's INS cannot provide accurate positioning (within 10 meters) after extended periods (hours to days) underwater (Goward, 2022). Particularly for vehicles moving in all three axes like a submarine, providing positioning information with 200-meter accuracy after only a few hours is difficult (Goward, 2022). Civilian-rated INS can experience error growth at a rate of one nautical-mile per hour without outside reference (Schmidt, 2015). Depending on the system's quality and initial alignment, a submerged submarine equipped with a military-grade INS can experience position drift of one nautical mile after anywhere between one to 15 days, although their exact performance ratings are guarded secrets (Hess, 2015; iXblue, 2024; Marks, 2014; Reynolds, 2014). Information from navigation aids and onboard sensors, like pressure depth sensors, can and should be integrated with the INS to reduce the accumulation of error/uncertainty in the EP. Robogate (2018) demonstrates the opportunity of using measured gravity gradients with a digital gravity gradient map to further increase the INS solution's accuracy, which could allow a submarine to remain submerge for longer periods of time. While a submarine's exact INS drift over time is classified, periodic access to an external accurate and reliable positioning source is essential for safe submarine navigation (Goward, 2022).

While underwater, the accumulation of positioning error is unbounded as seen in Figure 8, demanding periodic position updates from an external source (Hess, 2015). Although relatively noisy and susceptible to interference, GNSS is the forerunner for INS position updates since it provides worldwide, low-cost, high-accuracy position estimates with no long-term drift (Schmidt, 2010; Schmidt & Phillips, 2010)[2]. The worldwide coverage of GPS provides a submarine with accurate positioning information to update its INS anywhere in the world, in any weather, or time of day, allowing it to safely dive again and continue its mission.

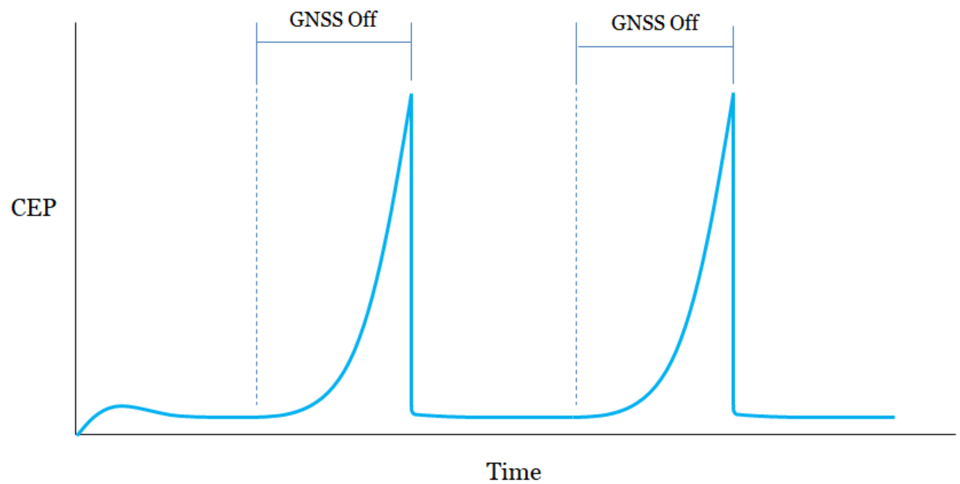

Figure 1. The accuracy of an integrated GPS/INS, which experiences a loss in accuracy without GNSS connectivity. These results were seen in Hess (2015) where submarine INS accuracy was verified with GNSS outages. Image adapted from Grewal (2020).

In addition to providing continued navigation data in a GPS-denied environment, an integrated GPS/INS system can help identify poor GPS positioning data before that data can cause errors in other systems. The GPS and INS are integrated via a Kalman filter, a recursive mathematical technique used to compute the optimal (most likely) solution of two estimates though weighted values (Marvel, 1998; Robogate, 2018). This error state vector tells the system how to best correct errors and update the navigation solution’s variables (Marvel, 1998). In the submarine’s case, multiple GPS measurements are processed by the Kalman

[2] A GPS update does not re-align the INS. The GPS data only provides a position correction for the GPS/INS systems output.

filter to update the INS before diving again (Schmidt, 2015). As the system works through time with many measurements, the both GPS and INS errors will be modelled more accurately, continuously improving the system's position estimate with better accuracy than if either system were working independently (Hess, 2015; Marvel, 1998; Schmidt, 2015; Schmidt & Phillips, 2010).

With regards to integrated systems, the use of Kalman filters is the most popular design for position estimation techniques due to its effectiveness in most situations, ease of implementation, and low computational demand (Cahyadi, Asfihani, Madriyanto, & Erfianti, 2022). A major design decision in integrating a GPS receiver with an INS is the location of the Kalman filter with respect to the integration. Tightly-coupled architectures (Figure 9) use raw data to help produce a solution, and their operation with less than four satellites' signals make them applicable in GNSS-challenged environments like urban canyons (Marvel, 1998). The system is optimized to use however many satellites are available (Schmidt & Phillips, 2010); even the used of one satellite can benefit an integrated solution (Marvel, 1998). Altogether, tightly-coupled set-ups can provide better navigation performance and are more reliable under high dynamics (Schmidt & Phillips, 2010). However, tightly-coupled architectures are more complex to develop and require more processing power, making them more costly to implement.[3]

[3] Tightly-coupled architectures are commonly seen with vehicles expecting short-term GNSS outages or extensive multipath error, such as UAVs or autonomous vehicles navigating urban centers.

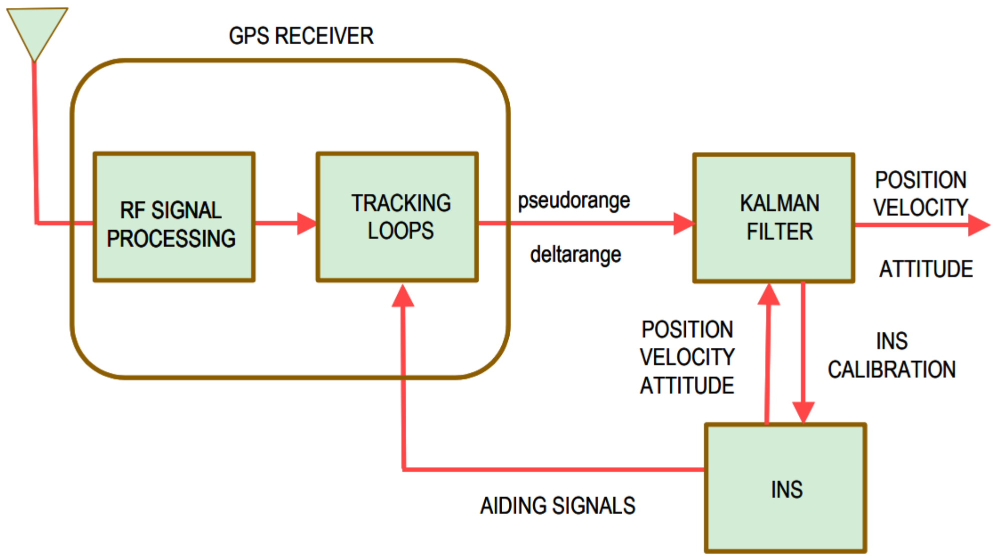

Figure 2. A tightly-coupled architecture sending raw GPS data to the Kalman filter (Schmidt, 2015).

Alternatively, in a loosely-coupled architecture (Figure 10) the GPS and INS operate independently, where the GPS data is filtered by the receiver's own Kalman filter before being sent to the through a master Kalman filter to the INS[4] (Marvel, 1998; Noureldin, Karamat & Georgy, 2012). Instead of using raw/unprocessed sensor data, a loosely-coupled GPS/INS requires that the GPS receiver output three-dimensional navigation solution, requiring at least four satellites. The receiver's Kalman filter helps estimate position, velocity, and acceleration errors as well as clock bias and clock drift, and ensures the measurements meet the integrated system's requirements (Boguspayev *et al*., 2023; Schmidt & Phillips, 2010). Only when the incoming GPS data meets the INS accuracy threshold does the receiver's Kalman filter creates a state vector which is sent to the master Kalman filter and then the inertial measurement unit (IMU) to correct the INS error (Boguspayev *et al*., 2023; Cahyadi, Asfihani, Madriyanto, & Erfianti, 2022; Marvel, 1998; Wang & Walter, 2023). In the event the submarine is forced to dive unexpectedly, either to avoid a collision or avoid being detected, the Kalman filter prevents inaccurate and incorrect positioning data to be forwarded to

[4] The use of two separate Kalman filters is referred to as a "decentralized" configuration (Noureldin, Karamat & Georgy, 2012).

the INS (Hess, 2015)[5]. The only INS information fed back to the receiver is position for the purpose of aiding in satellite signal acquisition, enabling the code generator and oscillator can make better-informed estimates of the incoming signals' frequencies and code phases (Grewal, 2020; Schmidt & Phillips, 2010).

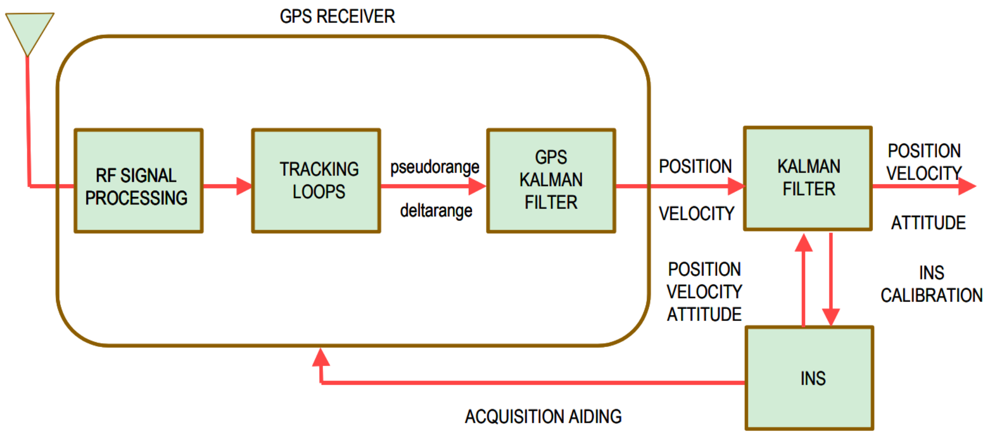

Figure 3. A loosely-coupled architecture where only processed and valid GPS information is passed onto the INS and other navigation systems (Schmidt, 2015).

A major benefit of loosely-coupled architectures is that they can be based on existing, commercially-available GPS receivers and INS, making them popular in the outfitting of military vehicles like submarines (Boguspayev *et al*., 2023). In terms of GNSS data, a drawback of the loosely-coupled architecture is that the system filters the GNSS data- some of which could be used by the Kalman filter to improve the overall navigation solution. Even if the GPS solution is considered accurate, one drawback though is that any errors in the GPS solution (due to any combination of factors discussed in Chapter 1) will be passed through the filters onto the INS and end-users.

On the surface with satellite coverage, the calibrated Kalman filter predicts the INS error, which is subtracted from the INS output, resulting in accurate position estimates (Boguspayev *et al*., 2023). The loosely-coupled architecture cannot conduct GPS-aided navigation when its number of available satellites is fewer than the minimum four (Noureldin, Karamat & Georgy, 2012). Thus when GPS connectivity is inevitably lost upon the submarine diving, the INS of the integrated system works as a standalone system with its inherent accumulation of error (Boguspayev *et al*., 2023). Unavailability of GNSS signals with integrated

[5] Hess (2015) examines the consequences of poor GNSS positioning fed into a submarine's INS.

GPS/INS is a main focus of current research, since it impacts how autonomous land and air vehicles perform (Boguspayev *et al*., 2023); however, the loss of GNSS signals is fundamental in submarine navigation. The accumulated INS error, or POE, is expected and can only be nulled upon a good GPS fix upon returning to the surface. In order to remain stealthy, it is in the submarine's best interest to get a good fix – one that meets the Kalman filter's limits – as quickly as possible. Luckily for the submarine, the factors that usually jeopardize GNSS measurement integrity and delay fixing times, including multipath, the environment, and signal interference are essentially zero in the open ocean.

1.2 Submarines and GPS

When surfaced or at periscope depth (PD), GPS information updates the INS position and is distributed to the end-users. Diving the submarine is an inherent evolution of the submariner. Unfortunately in doing so, the submarine loses all satellite connectivity due to the intervening water between the antenna and the satellite which absorbs the electromagnetic radiation that makes up GPS signals. The loss of GPS connectivity forces the submarine INS to navigate as a standalone system despite an inevitable accumulation of error while underwater (Fu & Lv, 2021).

Figure 5 outlines the differentiation between a surfaced submarine, a submarine at PD, and a dived submarine which is out of GNSS connectivity. For safe navigation on the surface or at PD (Figure 6), GPS position is displayed on SHINNADS due to its higher assigned priority than INS. Maritime testing demonstrates that at a depth of 0.5 centimeters, a GPS antenna can acquire GPS satellites but cannot calculate a position fix, while no satellites are visible at a depth of one centimeter and below (Griesel, 2006). At that point, the submarine finds itself in a GNSS-denied environment, where the INS is the sole provider of positioning data to the submarine's systems (Northrop Grumman, 2023).

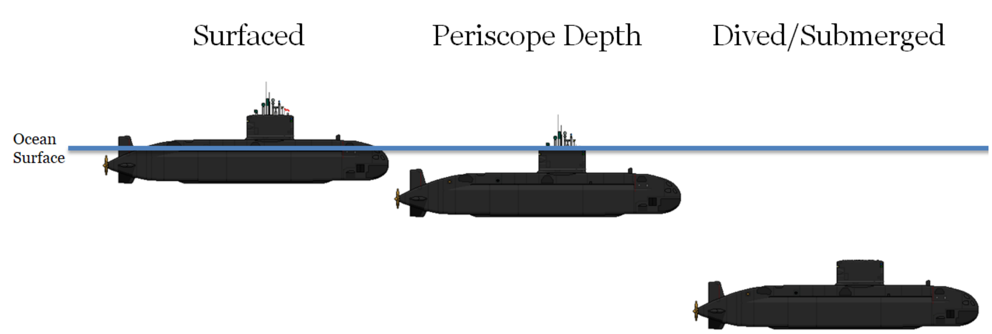

Figure 4. Differentiation between a surfaced submarine, a submarine at periscope depth, and a dived submarine which is out of GNSS connectivity.

As soon as GPS connectivity is lost upon diving, the last valid position and time information is stored in the GPS receiver's memory along with almanac, ephemeris, and user settings (Griesel, 2006; Measurement Systems Limited, 2023). Highly-accurate atomic clocks onboard the submarine maintain timing information for system synchronization, but also supplements the loss of GPS time while underwater. With its last known GPS fix information, the INS becomes the primary source of positioning data through dead reckoning. The INS maintains an independent navigation picture with the submarine's position and velocity by accounting for all movement since an initial position, and also calculates attitude. That data is distributed to end-users for their continued functionality (Rogobete, 2018).

The combination of alignment, sensor, and computation error results in increasing positioning error over time while underwater. Although the total positioning uncertainty may be tolerable in the short-term, that uncertainty grows to unacceptable sizes in the long-term, calling for correction by an external positioning source (Rogobete, 2018). By using GPS for that correction, the inertial sensor error that has accumulated since the last GPS position update cannot be eliminated until the next successful GPS fix (Rogobete, 2018). The accuracy of those GPS fixes plays a role in the accuracy of the INS solution when the INS has to operate as a standalone system (Hess, 2015).

The total error (or uncertainty) in the submarine's EP is represented on SHINNADS by a circular or elliptically-shaped Pool of Errors (POE) as seen in Figure 7, which takes all sensor uncertainty, system bias, and positioning inaccuracy into account (Rogobete, 2018). Once out of GPS coverage, the POE's major and minor radii increase in size with the submarine's time underwater, along with its speed and maneuvers, where there is an equal probability the submarine is anywhere within the POE boundary (Keller, 2008; Keller, 2023). Fortunately, submarines are low-dynamic vehicles which do not conduct high-G turns or sudden velocity changes, which are known to degrade INS accuracy (Hess, 2015). While underwater, operators then use every opportunity available to reduce the POE (depth area for example) which maximizes the time the submarine can safely remain submerged. When the POE is too great that it inhibits and/or jeopardizes the submarine's movements, it requires a GPS position fix to update its position and null the accumulated error (Marvel, 1998; Payne, 2010; Rogobete, 2018).

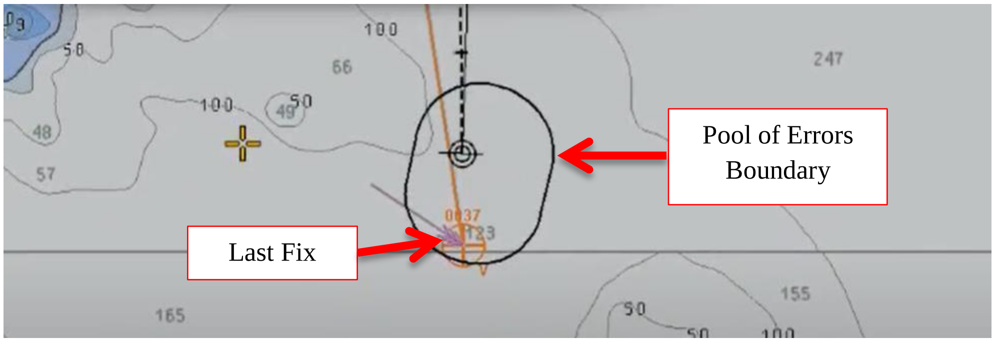

Figure 5. A submarine's POE as it would appear on the SHINNADS. Due to a loss of GPS connectivity and total sensor error, the submarine has an equal probability of being anywhere inside the POE boundary.

For strategic and tactical purposes, a submarine will remain submerged as much as possible. While this preserves its covertness, it will experience POE growth which may jeopardize safe navigation. The accumulation of positioning error over time while underwater makes the submarine's EP increasingly inaccurate if not updated with a GPS fix to reset the vessel's position (Cole, 2015; Keller, 2023). Even military- and strategic-grade INS, like those fitted onboard warships and submarines, cannot provide accurate indefinite positioning data without outside reset (Goward, 2022). In order to regain connectivity to nullify the INS error, at least one mast must be raised above the surface of the water to re-establish satellite connectivity. This penetration of the water surface can reveal the submarine's position to sensors on nearby ships or submarines as well as create a wake that can be seen by aircraft or even satellites, which can all compromise the submarine's covertness. Thus if no other surfaced or PD evolution are planned, the acquiring of a GPS fix should be done as quickly as possible.

Figure 6. A submarine at periscope depth, where masts can be raised above the surface of the water for a number of purposes, including GNSS positioning (Airbus, 2024). However, masts can be detected by ships and aircraft, while the resultant wake can be visually

When needed in open waters, submarines can spend extended periods at PD or on the surface, allowing for considerable time for satellite acquisition and subsequent fixing in conjunction with other evolutions[6]. In coastal waters on the other hand, and especially in an adversary's territorial waters, a submarine spends limited time at or near the surface of the water to avoid detection (Hess, 2015). Exposing either the search periscope or communications mast above the surface of the water for a GPS fix for any amount of time is unfavourable. Thus the time required between GPS antenna exposure and the first good fix is of prime importance in maintaining the submarine's covertness. Any time the antenna spends above the water unnecessarily exposes the submarine and the crew to sensors on ships, satellites, and aircraft- the latter being the majority source of surfaces submarine detection, particularly during calm sea conditions (Hess, 2015; Payne, 2010). Further to the presentation of a target for sensors, a submarine on the surface also produces more noise which is also detectable by those same ships, other submarines, and underwater sensors. A submarine on the surface is also both

[6] Examples include satellite communications, domestic routines, charging air bottles, and battery charging (for diesel-electric submarines).

slower and less maneuverable. Altogether, it is in the submarine's best interest to get a good GPS fix as quickly as possible, update the navigation systems, and dive to deep waters.

Once the submarine breaches the surface with its GPS antenna the receiver can re-establish GPS connectivity. Unlike a warship though whose GPS antenna is continuously within GPS connectivity and its receiver continuously calculating its position, a submarine returning from a dive must re-acquire the GPS satellites and download the necessary navigation data before calculating a first position fix. With a non-volatile memory (NVM), almanac data can be stored for quick reference, thus saving time in how much navigation data is needed form the satellites. Equipped with atomic clocks for precise time-keeping onboard, submarines can acquire the P code without first processing the C/A code through time aiding, expediting the time required to fix the submarine's position (AGARD, 1988). Since the precise time is known, the receiver can calculate its three-dimensional position off only three satellites, again expediting the time required to fix (AGARD, 1988). The use of dual-frequency receivers within the RCN for its increased positioning and signal security factors also helps reduce time to first fix (TTFF) (Cao, 2008). However, the very first fix by a GNSS receiver is usually inaccurate. After the first fix, the receiver will refine its position estimate as more satellites are acquired and data downloaded until it converges towards the INS's accuracy threshold - a limit that prevents the acceptance of inaccurate positioning information which can degrade the submarine's potential. Only until the incoming positioning information reaches that accuracy threshold is the data accepted, the navigation systems updated, and the submarine can safely re-submerge if needed. While not as long as the convergence time for Precise Point Positioning (PPP), the time required to re-acquiring satellites, access the require satellite information, and compute a position estimate can be in the range of minutes (Fu & Lv, 2021; Measurement Systems Limited, 2023). An extended wait time is not efficient for a platform whose survivability rests on remaining covert and undetected. The less time a submarine has to expose any of its masts above the water for a GPS fix, the easier it is for the submarine to remain undetected (Marks, 2014).

1.2 Statement of Deficiency

There is no shortage of studies examining positioning accuracy of Global Navigation Satellite Systems (GNSS) in different environments such as urban canyons, particularly as the usage of satellite positioning solutions expands to new applications, including current developments in autonomous vehicles. Furthermore, research follows the understanding that the vast majority of GNSS users either have their receivers permanently operating or can afford to wait a few minutes for good positioning. Although self-driving cars are expected to encounter loses in GNSS connectivity, those gaps are expected to be limited in duration. Studies have shown the enhanced performance of multi-GNSS receivers, with shorter re-acquisition times and better positioning accuracy; however, those studies do not analyze the quality of those first fixes or the convergence rate of the solution (Anghileri *et al*., 2008; Cao, 2008; LeVeel, n.d.; Luo, Chen & Richter, 2017).

Where there are gaps in literature are GNSS applications with longer periods of intermittent connectivity, such as that experienced by submarines. The analysis of GNSS positioning accuracy in remote environments away from augmentation services which strive to improve satellite positioning for as many users as possible, is also limited.

1.3 Objective

The introduction of newer GNSS and augmentation systems has the potential to enhance positioning services for the RCN and its submarines. One possible enhancement is the reduction in the time required to reach the fix accuracy threshold, allowing a submarine to update its navigation systems and re-submerge after less wait time either at PD or surfaced. This thesis will investigate if a combined GPS+Galileo receiver could reduce the wait time from antenna exposure to the first accurate fix that can be used for navigation. Less time spent at the surface would reduce the submarine's risk of being detected by naval, aerial, and satellite-based sensors, promoting its survivability. A receiver that gets an accurate fix faster than the currently equipped GPS-only receivers would add value to the VICTORIA Class submarines.

In replicating the submarine's conditions, the performance of a GPS-only receiver and a GPS+Galileo receiver will be compared to assess if a multi-GNSS receiver can calculate a position fix that meets the submarine's accuracy threshold faster than the a single-GNSS receiver. This thesis is unique in that it explores the two circumstances of intermittent GPS connectivity and the remote maritime environment together.

1.4 Organization

The structure of the thesis is as follows. In Chapter 2, a literature review is presented, focusing on time for GPS fixing and the factors involved in both TTFF and the time to the first fix that meets the submarine's accuracy threshold. Next, methods currently employed to reduce GPS fixing times are reviewed along with an assessment whether they can be applied to submarine positioning in remote maritime environments. In Chapter 3, the methodology for data collection in comparing receivers is outlined. In Chapter 4, the results of the data collection are presented. In Chapter 5 contains a statistical analysis of the results. Chapter 6 discusses the results and their application not only in submarine warfare but also to the general GNSS community, as well as recommendations for future work. Finally, in Chapter 7, the findings of this research are summarized.

Chapter 2

Literature Review

In this chapter, we explore the many factors that influence Time to First Fix and its role in GNSS receiver performance. However, a more noteworthy parameter of receiver performance, in particular for submarines, is the wait time for the first "good fix" which is the first position estimate that can be used for safe navigation and INS updates. Discussed in this literature review are the factors that determine TTFF and options to reduce TTFF. The new Galileo GNSS is explored in more depth as the prime candidate to reduce the time for a submarine to obtain its first valuable position estimate upon returning to the surface.

2.1 Early Satellite Navigation

In 1964, the United States (U.S.) began operating the Navy Navigation Satellite System (NNSS) Transit, the first practical application of satellite navigation (Tewelow, 2020). Comprised of a series of five polar-orbiting satellites (with additional spare satellites), NNSS assisted U.S. Navy ships and submarines in determining their positions. While ships could calculate their position with an accuracy of 200-meters, encrypted signals enabled submarines to achieve a superior accuracy of 20-meters (Tewelow, 2020). However, the limited number of satellites resulted in extended fixing times, sometimes up to one hour or more depending on where the nearest satellite was relative to the submarine (Lawrence *et al*., 2017; Tewelow, 2020). Additionally, positioning off a single satellite meant that a submarine had to remain stationary for 10 to 16 minutes to allow for geometric diversity calculations in the satellite's rate of change (Lawrence *et al*., 2017; Kumar & Moore, 2002).

Positioning accuracy was refined in the late 1960s following the launches of two Timation satellites equipped with atomic clocks, which maintained and broadcasted a time reference signal (Moore, 2002; Tewelow, 2020). The integration of atomic clocks into the equation enabled receivers to calculate the time of flight (TOF) of satellite signals - the difference between the time the signal left the satellite and its time of arrival at the receiver (Kumar & Moore, 2002), enabling receivers to be mobile when calculating their positions.

Into the 1970s, advancements in electronics enabled NNSS to evolve further, eventually becoming accessible to non-military users such as surveyors. Although position determination was slow and the system's accuracy was poor, NNSS laid the foundation for more complex, dependable, and consistent GNSS (Kumar & Moore, 2002). By embedding atomic clocks onboard the satellites and instituting more precise time-keeping techniques, orbital tracking, and ground control measures, a GNSS of sufficient number of satellites could provide fast, reliable, and accurate positioning anywhere on Earth.

In modern day, positioning a submarine off the NNSS has yielded to more complex GNSS. The time required for fixing is now significantly abbreviated and a submarine does not have to be stationary to calculate its position. Still, the concept of ranging off satellites moving across the sky remains the same.

2.2 Global Navigation Satellite Systems

GNSS are a form of one-way radio navigation where satellites transmit low-power radio frequency signals consisting of their position and the system's time (Grewal, Andrews, and Bartone, 2020; Kumar & Moore, 2002). In a three segment configuration, a GNSS requires a dedicated space segment consisting of satellites, a ground segment featuring control and tracking stations, and the user segment comprised of the individual receivers.

Each satellite houses redundant, highly precise atomic clocks that are synchronized to a common time reference, providing the basis for their broadcasts (Kumar & Moore, 2002). Along with the satellite time, the satellites transmit a pseudorandom code with a navigation data message containing all the information needed by a receiver to estimate its position (Anghileri, 2013). While positioning accuracy depends on the quality of the data message's content, GNSS capacity[7], robustness, and timeliness have to do with the message's design (Anghileri, 2013). Following the principle of a uniform velocity of electromagnetic radiation through a medium, a receiver calculates the instantaneous range (pseudorange) to each satellite (*d*) by calculating the time of flight (TOF) (*T*) of the satellite signal:

$$d = cT \tag{2}$$

where *c* is the speed of transmission (Carlson, Crilly, and Rutledge, 2002).

Through multilateration, involving the comparison of multiple time stamps from at least four satellites within its line-of-sight (LOS), receivers can calculate their three-dimensional position relative to the satellite constellation in real time. GNSS time also becomes available through position fixing, and receiver velocity can be determined through subsequent fixes or more accurately through satellite Doppler (Kumar & Moore, 2002), (Payne, 2010). If more than four satellites are within the receiver's LOS and the receiver's circuitry is adequate to track them, the solution becomes over-determined, leading to a more accurate position estimate.

[7] Related to the effective bit rate, or the efficiency factor (Anghileri, 2013).

The position can then be converted into a known frame of reference, including latitude, longitude, and altitude for navigation purposes.

There are currently four operational GNSS[8]: the American Global Positioning System (GPS), the Russian Global Navigation Satellite System (GLONASS), the Chinese BeiDou, and the European Union's Galileo. As the most senior system, GPS is arguably the most developed and popular GNSS (Schmidt, 2015). However, GPS does not necessarily offer the best positioning performance. Tests by Liu *et al*. (2022) revealed that when only considering core-GNSS satellites, Galileo offered the best space signal positioning accuracy, followed by BeiDou, GPS, and finally GLONASS.

While receivers specific to one constellation (e.g. a GPS receiver) can only receive signals from that one constellation, multi-GNSS receivers are capable of interpreting signals from multiple constellations simultaneously. Multi-GNSS receivers have been the subject of multiple studies, whose results show improved positioning accuracy, better positioning reliability, and shorter solution convergence times. Although positioning and timing services may be accessible from each of these systems, their positioning performance and accessibility are subject to their governing bodies and may be scaled back without notice.

2.3 Global Positioning System

GPS, originally called Navigation Satellite Timing and Ranging (NAVSTAR) GPS, is controlled and maintained by the U.S. Department of Defense (DoD). Its three-dimensional positioning and timing services are available free-of-charge around the world. Following the success of the NNSS program, the U.S. DoD began research and development (R&D) on the NAVSTAR program and its first satellites in 1973 with the launch of the first four satellites in 1978 (Kumar & Moore, 2002). Most importantly, the new constellation would consist of more satellites than NNSS so a receiver anywhere on Earth would have access to multiple satellites simultaneously, increasing signal availability reducing the wait time for a more accurate position estimate (Kumar & Moore, 2002).

Today, the GPS network of at least 24 satellites (including in-orbit spares) in Medium Earth Orbit (MEO) provides 24-hour, all-weather, high-accuracy, worldwide three-dimensional positioning and timing services to users with GPS receivers (Grewal, Andrews, and Bartone, 2020). Organized in six orbital planes with orbital inclinations of 55 degrees and orbital periods of 12 hours, the constellation is evenly spread out as to provide coverage over the entire planet

[8] Both the Indian Regional Navigation Satellite Systems (IRNSS) and Japanese Quasi-Zenith Satellite System (QZSS) are only regional systems. GNSS interoperability could result in difference performance metrics within their areas of coverage.

(Kumar & Moore, 2002). Due to the number of satellites and their relative spacing in orbit, the GPS constellation provides overlapping satellite footprints, where GPS receivers anywhere on Earth are subjected to the GPS broadcast from at least six satellites at any time. Although the satellites provide coverage of the entire planet, there are some areas (like higher latitudes) that see fewer satellites in the sky than others and thus experience variations in positioning accuracy (Hunt, 2020).

A global network of ground stations also controlled by the U.S. DoD, in conjunction with a master control station in Colorado, maintain the GPS satellite constellation and provide continuous updates to the satellites to maintain system accuracy (Kumar & Moore, 2002). The position and orbital motion of each satellite are monitored, along with their individual health and clock status/time. Any updates and correction information is transmitted to the satellites via separate uplink stations (Kumar & Moore, 2002). With spare satellites in orbit to maintain continuous coverage, a satellite can be taken offline for any reason with the other satellites instructing receivers to ignore that one satellite's signals. The GPS Block III satellites being launched at the time of this writing will further augment positioning accuracy with greater signal integrity with new Search and Rescue (SAR) capability. Furthermore, satellite cross-links on Block III satellites will enable high-speed data sharing between satellites in orbit (Schmidt, 2015).

The satellite broadcast contains all the information needed by the receiver to estimate its position, strategically formatting within a navigation data message. Almanac data contains the whole constellation's rough orbital information and status, enabling a GPS receiver to know which satellites are both overhead and healthy, and thus should be used for positioning (Kumar & Moore, 2002; Measurement Systems Limited, 2023). Ephemeris data is specific to each satellite, containing precise orbital and clock correction data for that satellite, and is only downloaded once a GPS receiver locks onto each satellite (Measurement Systems Limited, 2023). Both almanac and ephemeris data are continuously updated through ground monitoring- ephemeris data is updated every two hours but remains valid for up to six hours, while almanac data is updated every six days (minimum) but remains valid for up to 180 days. Together, the ephemeris and clock information is the minimum information required by a receiver to estimate its position (Pena *et al*., 2009; Samson, 2011).

All operational GPS satellites continuously broadcast the navigation data messages on multiple carrier frequencies (L1C/A 1575.42 MHz, L2C 1227.6 MHz, and L5 1176 MHz). While some GPS signals are civil signals and thus widely available (L1 C/A, L1C, L2C, and L5), others are dedicated military signals (L1/L2 P(Y) and L1/L2M) which require a controlled decryption module. The introduction of the L5 frequency among all operational GPS satellites, anticipated for the year 2027, is expected to further increase positioning accuracy. Furthermore, the Block

IIIC's new M-Code will further provide signal security and anti-jam capabilities due to its higher signal power (20 dBW above current signal levels) (Schmidt, 2015). The higher power of M-Code, with its specific spot beams, will enhance anti-signal jamming strategies specifically for military operation areas (Schmidt, 2015).

The GPS navigation data message, comprising all navigation and clock data required for positioning as well as that satellite's identifying pseudorandom noise (PRN) code, is 37,500 bits long and is repeatedly transmitted at a rate of 50 bits per second on the L1 C/A code (Walter, Gunning & Blanch, 2016). While ephemeris and clock data is repeated every 30 seconds, the entire almanac requires 25 consecutive messages and 750 seconds (12.5 minutes) to be transmitted in its entirety. The 30-second message is divided into five sub-frames as seen in Figure 1 (Walter, Gunning & Blanch, 2016):

a) Sub-frame 1: Satellite identification, health bits, and time information;
b) Sub-frame 2-3: Ephemeris data; and
c) Sub-frame 4-5: Part of the Almanac data.

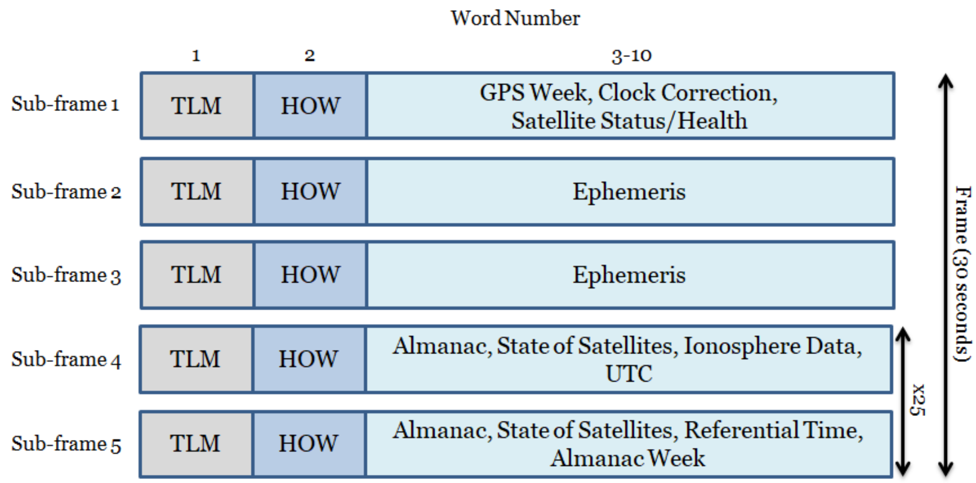

Figure 7. The GPS Navigation Data Message structure.

The GPS time, clock correction, and ephemerides are front-loaded in the GPS data message and repeated every 30 seconds. If a receiver has the almanac data either from a previous fix or from assist data, all the remaining information it needs to estimate position lies within the first three sub-frames which last 18 seconds (Yang, 2018). If any of this data is interrupted, the receiver must wait for the next 30-second transmission to complete the download. In GNSS-challenging environments, this can lead to first fixes taking multiple minutes, which is undesirable in an emergency (Yang, 2018).

GPS is a low-power system, with satellites transmitting their broadcasts at 27 Watts (44.3 dBW). By the time the signals reach the surface of the Earth from MEO, they have been significantly reduced to approximately 1.6 x10^{-16} Watts (-158 dBW) due to spreading loss (Lawrence *et al*., 2017; Schmidt, 2015). The system's low power results in the navigation data message transmission being slow, requiring the 12.5 minutes to be fully broadcast as previously mentioned (Kumar & Moore, 2002). The system's low power results in not only an extended data message, but also results in the satellite signals being highly attenuated by the time they reach receivers on Earth- particularly in obscured environments (Lawrence *et al*., 2017; Xu *et al*., 2024). By the time they transit the atmosphere and join all other electromagnetic radiation near the surface, they are ultimately buried in background noise (Kumar & Moore, 2002). Through spread-spectrum communication technology, receivers are required to amplify the signal in order for it to be recovered and used to calculate a PNT solution (Kumar & Moore, 2002).

2.4 GPS Receivers

Users require a GPS receiver in order to take advantage of the system's worldwide availability. Keeping mass-market receivers and their technology at a relatively low cost enables widespread usage and worldwide adoption of GPS positioning, which has become synonymous with navigation, particularly in personal vehicles, mobile devices (Larson & Wertz, 2005), and recently autonomous vehicles (Loizou, 2020). Once initialized/powered-on, the receiver will passively maintain connectivity with visible satellite and maintain a position estimate.

The performance of a receiver, including its accuracy and fixing time, is related to its quality and cost. Consumer-grade receivers are mass-produced and are becoming more user-friendly and increasingly smarter with the advancements in electronics and processing (Linty, 2014). With their cost remaining relatively low, their increasing capability paired with decreasing size is enabling more location-based services (LBS) and applications throughout society (Kumar & Moore, 2002). Features that drive up receiver cost are increased number of channels (allowing for simultaneous tracking of multiple satellites), multi-constellation capability, complex models and algorithms to compensate for predictable errors, and high-grade antennas that can detect and track even the weakest of GNSS signals. Professional-grade receivers implement the above mentioned capabilities amongst other more complex strategies to output the best possible solution, usually required for surveying and high-precision applications. However, the speed and capability of receiver technology cannot compensate for the GPS system's long data messages which are a result of the system's low power (Kumar & Moore, 2002).

When a GPS receiver is powered-on, it runs a search space to find GPS satellite within its field of view (FOV) (Anghileri *et al*., 2008). Since the satellites are in constant motion around the Earth, the same satellites are not always visible, and so the receiver must assess what signals it is seeing and filter out unwanted noise. In a worst-case scenario, when the receiver has no information on which satellites are overhead, it searches over the entire Doppler-shifted frequencies of all orbiting GPS satellites. Any information the receiver can have ahead of time on satellites or location can reduce the number of satellites or reduce the frequency bands to be searched (Wells, 1987). When powered on for the first time, the receiver requires a full almanac download for all data pertaining to the constellation. Being common to all GPS satellites, the almanac can be downloaded from any GPS satellite. Once the antenna achieves lock-on with any GPS satellite, the receiver can start downloading the navigation data required to compute a position estimate. Storing previously downloaded navigation data in NVM means that subsequent initializations do not always require the full almanac download in order to provide a first position estimate (as long as that data remains valid), significantly reducing the search space and thus time needed to acquire satellites and provide a position estimate (Anghileri, 2013; Grewal, 2020; Kumar & Moore, 2002). Ephemerides and GPS clock information on the other hand are always required for positioning. With the satellites continuously transmitting their data on a loop, the receiver's antenna requires continued connectivity with the satellites; if interrupted, the receiver must wait for that message to be repeated in order to download the missing data. Once the all the required navigation data is received, demodulated, and decoded, that satellite can be used for positioning.

The power consumption of a receiver in its initial satellite signal acquisition and data downloads are non-negligible, putting high power demands on receiver batteries (Linty, 2014). From its initial position estimate, the receiver continuously determines its position- if additional channels are not required, they can be shut down to save power. Receivers in urban environments, which may experience LOS interruptions due to structures, may experience extended positioning times due to breaks in the almanac download, requiring extended download times where multiple channels operating consume battery power. Thus the most likely satellites to be used for positioning are those with high elevation angles relative to the receiver (Samson, 2011). Repeated breaks in satellite LOS while downloading data can even result in a receiver's complete failure to estimate position. Thus the storage of almanac data in NVM saves power in the long run, as long download times are not required upon initialization.

The receiver conducts the TOF calculation of each satellite signal to which it is connected. However, the high transmission speed of electromagnetic radiation demands nano-second timing capability to accurately measure each signal's time of arrival. The precise time-keeping of the satellite clocks is critical- if one satellite is misaligned by one millisecond, a positioning error of 300 kilometers can occur (Kumar & Moore, 2002). To limit the cost and complexity of GPS receivers, they contain inexpensive timing devices that are synchronized with GPS time by a fourth satellite while the first three generate the three-dimensional position fix (Kumar & Moore, 2002; Larson & Wertz, 2005).

Through the time delay in at least four satellites' identifying PRN code and the accompanying almanac and ephemeris data at the time of signal transmission, receivers can estimate their three-dimensional position (x, y, and z) relative to the GPS constellation as well obtain GPS time (UTC) based on the intersection of the four time signal spheres (Kumar & Moore, 2002). Once position is estimated, receivers consider any corrections broadcast in the satellite data messages, along with internal algorithms that adjust for predictable errors, such as atmospheric effects (Kumar & Moore, 2002). The estimated position can then be converted to useable latitude and longitude coordinates with altitude above sea level on the ellipsoidal model of the Earth (WGS84). Instead of calculating receiver direction of travel and speed by differentiating the change in position over time, receivers use the Doppler shift in the satellites' signals (Kumar & Moore, 2002). Exploiting the Doppler shift is the preferred manner over the time interval due to the noise in GPS signals, which is not a factor in their Doppler shift (Kumar & Moore, 2002).

If the receiver's hardware suffices, tracking additional visible satellites beyond the minimum four enables more accurate positioning. As long as a receiver has direct LOS with GPS satellites, the receiver's number of channels dictates how many satellites it can download ephemeris and timing information from at a single time. Five-channel GPS receivers allow for four channels to solve for three unknowns in addition to time. A fifth channel can measure ionospheric delay and keep track of satellite movement to select the optimal next satellite to acquire near the horizon (AGARD, 1988). Multiple independent channels to simultaneously process satellite data increases error detection to enhance receiver performance. This receiver structure also enables Receiver Autonomous Integrity Monitoring (RAIM) which assesses each GPS signal, can isolate faulty measurements, and alert the user of accuracy discrepancies (Schmidt, 2015). GPS receivers with at least a dozen channels offer users the highest positioning performance by averaging all the TOF of all visible satellites. Military-grade receivers, which have many channels and also de-crypt protected GPS signals, are installed on warships, submarines, and high-dynamic vehicles like fighter aircraft to offer even greater positioning accuracy and added anti-jamming capabilities (AGARD, 1988).

A number of environmental factors affect the receiver's ability to lock onto satellites and download the required navigation data in a timely manner. Nearby telecommunications or RADAR using similar frequency bands can cause unintentional interference, forcing the receiver to distinguish between GPS and non-GPS signals (considered noise), which can take time (Schmidt, 2015). Additionally, GPS signals can be attenuated by nearby infrastructure, snow, ice, mountains, and even trees that reduce the signal-to-noise ratio and result in low data quality and ultimately poor positioning performance (Luo, Chen & Richter, 2017). Antenna orientation, movement, and even nearby cabling can also negatively impact signal processing and delay positioning calculations.

There are also a variety of interference strategies whose goal is to disrupt a user's ability to use GPS for any PNT application. The low power of GPS signals makes them vulnerable to jamming, where a nearby device can emit similar signals but at a higher power, preventing satellite acquisition. Spoofing can also be used to trick a receiver with false information (Schmidt, 2015).

It is a common misconception that GPS receivers (particularly those within smartphones) rely on cellular data to determine position (Garmin, n.d.c). The misunderstanding stems from devices requiring an internet connection to download the terrain or road maps on which to display their position estimate. GPS positioning is also one-way, in that devices do not share their position estimate with either the constellation or with other devices. If connected to another device over Bluetooth, only then may receivers share their positioning information which can be used to update personal contacts or even alert an emergency contact of their position. However, these separate devices require both a paid subscription for data exchange and that the device be within data coverage (Garmin, n.d.b). If operating outside of data coverage, users can opt for a third-party communication device, such as in-Reach satellite communicator, which enables the sharing of a receiver's GPS data under a subscription plan for real-time tracking and emergency support through the Iridium constellation (Garmin, n.d.b).

The massive demand of consumers for fast and accurate positioning leads manufacturers to develop methods of augmenting GNSS signals. For example, devices without GPS receivers (like WIFI-only tablets and older smartphones) which cannot download data from GPS satellites can estimate their rough position through nearby registered WIFI networks which know their own location (Garmin, n.d.d). An unregistered WIFI network in contrast, such as a mobile hotspot, as well as tethered devices, may not help generate an accurate position estimate (Garmin, n.d.d). If equipped to do so, a position estimate can also be obtained through triangulation with three or more cellular towers, which similarly know their own locations. At least three towers are required for triangulation, while pinging more than three towers can increase the position estimate's accuracy. Information shared from these same WIFI and cellular networks can similarly be exploited to reduce GPS fixing time.

2.5 GPS Performance

The positioning accuracy of a GNSS receiver is the difference between the receiver's estimated position and its true position (Rychlicki, Kasprzyk & Rosinski, 2020). Positioning accuracy of GNSS is limited by its low signal strength, the length and contents of its navigation data message, and errors in the signal tracking (Schmidt & Phillips, 2010).

Although four satellites are required to be in-view of the receiver for three-dimensional fixes, they must also be of sufficient angular separation for accurate positioning (Larson & Wertz, 2005). The preferred satellite arrangement of the four satellites from the antenna's perspective for three-dimensional positioning is one satellite directly overhead (the zenith) with the other three spread evenly above the horizon, forming a tetrahedron relative to the user (Hunt, 2020). However, satellites appearing too low on the horizon are more prone to atmospheric error, and a shadow effect can stem from urban buildings that reduce the number of available satellites within a receiver's field of view, degrading positioning performance.

The geometry of the satellites relative to the receiver can be quantified as Dilution of Precision (DOP). While Geometric DOP (GDOP) and Position DOP (PDOP) can be used to quantify the quality of three-dimensional fixes, Horizontal DOP (HDOP) is representative of the quality of a two-dimensional fix which is valuable in marine navigation. Satellites grouped together yield a high DOP, which is undesirable. Even if directly overhead, grouped satellites are bad for positioning accuracy by giving less accurate horizontal positioning solutions (Measurement Systems Limited, 2023). The preferred layout resulting in low DOP is an even spread of satellites across the sky. Since the orbit of GPS satellites is inclined at 55 degrees relative to the equator, DOP is more dependent on latitude than longitude, causing GDOP and PDOP to degrade as the user's latitude increases beyond 55 degrees latitude (Hunt, 2020). GNSS positioning accuracy and fix quality are thus a combination of DOP, satellite availability, and error correction (Rychlicki, Kasprzyk & Rosinski, 2020).

The performance statistics of current GNSS positioning accuracy varies according to the source, and is constantly changing due to constellation modernization and new receiver technologies. In the decades since the introduction of GPS, both the horizontal and vertical positioning accuracies have continuously improved. Commercially-available dual-frequency receivers generate positional accuracy between five and ten meters through the Standard Positioning Service (SPS) (US DoD, 2007b). Military receivers with decryption capability of the L1 and L2 P(Y) code enable reception of the Precise Positioning Service (PPS), reducing error and improving positioning performance to between two and nine meters (Schmidt, 2010; US DoD, 2007b). GPS Block III satellites are expected to improve military receiver accuracy from ten feet down to three feet (Schmidt, 2015). In addition to more precise positioning, the encryption of PPS receivers also provide both anti-spoofing and Fault Detection and Exclusion (FDE) to protect navigation systems against spoofing and system anomalies, respectively (Trimble, 2004). The U.S. DoD has made PPS receivers available to its military allies, including Canada and other NATO countries (Space News, 2008).

Manufacturers tailor GPS receiver technologies, and thus receiver capabilities and performance based on the user's requirements. While cheaper receivers can display basic positioning information, more advanced and costly receivers are required for high-precision applications including surveying and research. Increased receiver cost can equate to lower fixing times and increased positioning accuracy from novel signal processing techniques and algorithms. Most of which however are protected by patents and thus not discussed in open literature.

Within the user segment (the receivers), GPS performance can be measured in one of two manners: circular error probable (CEP) or two times the root mean square (2drms) (Marvel, 1998). CEP outlines the radius of a circle which encompasses a particular percentage (often 50%) of the position measurements. Alternatively, the root mean square method defines a circle whose radius is 2drms, where:

$$2drms = 2\sqrt{\sigma_x{}^2 + \sigma_y{}^2} \tag{3}$$

A deliberate degradation in GPS performance, known as Selective Availability (SA), was initially built into GPS to prevent adversaries from exploiting the system's free high precision PNT capabilities. Although SA was disabled in 2000, the capability of the U.S. DoD to degrade GPS performance without warning was a major driver in other nations developing their own independent satellite navigation system. A press release from the US DoD in 2007 announced that future satellite builds (GPS III) would no longer support SA, which would assure users of GPS's dependability and the U.S.'s commitment to high system performance (US DoD, 2007a).

2.6 Positioning Errors

GPS receivers, along with other GNSS receivers, are not perfect in calculating the user's true position. Errors can present themselves in any of the three GNSS segments. R&D into GNSS technologies continues to reduce these errors to enable more precise and more accurate PNT services. However, there are inherent errors in any satellite positioning that prevents perfect positioning (Siemuri *et al*., 2022). Some sources of positioning error, such as atmospheric and multipath errors, are more significant than others and are thus prioritized by correction techniques including models and algorithms.

The GPS ground stations are continuously monitoring the satellite clocks, satellite trajectory, and overall satellite health, updating the navigation data message as required to mitigate against satellite position, orbit, and clock errors to maximize the system's performance. The GPS even accounts for the effects of General Relativity due to the differences in relative speed between the satellites in space and receivers on Earth.

As the signals transit the atmosphere, they are subject to ionospheric effects, atmospheric disturbances, and signal refraction (Kumar & Moore, 2002). Variations in solar activity and its effects in Earth's ionosphere can cause scintillation, which degrades signal quality and can even lead to GNSS outages. Ionospheric delay errors can cause positioning errors as large as 30 meters at their peak in the afternoon and usually between three and six meters overnight (Marvel, 1998; Hadas, Kazmierski & Sosnica, 2019). Tropospheric delay errors, mostly due to the water vapor within the atmosphere, can total as much as 30 meters (Marvel, 1998). Both tropospheric and ionospheric errors can be reduced by approximately half through models programmed in the receiver (Hadas, Kazmierski & Sosnica, 2019; Schmidt, 2010; Yan & Yhang, 2022). Since the effect of these errors is dependent on their frequency, both errors can also be reduced by the more expensive dual-frequency receivers which simultaneously measure the pseudorange of both satellite frequencies, increasing positioning accuracy and augmenting system reliability (Kumar & Moore, 2002; Rychlicki, Kasprzyk & Rosinski, 2020; Schmidt, 2010; Wang & Walter, 2023).

Although satellites may be visible to the receiver, they may not be available or ideal for ranging. Signals from satellites that appear too low on the horizon are subject to increased atmospheric scattering as they propagate through more of the atmosphere and may also be interrupted by obstructions, which reduces positioning accuracy (Kumar & Moore, 2002; Measurement Systems Limited, 2023). Receivers are programmed to select the more ideal in-view satellites to reduce interruptions. Receivers are also set with elevation angle limits (or mask angles) usually set to between 10° to 15° to avoid excessive atmospheric error (Measurement Systems Limited, 2023).

GPS receivers are also prone to multipath error, which is a source of noise caused by the signal interference and reflection off buildings, walls, roads, and even snow (Schmidt & Phillips, 2010). Multipath errors are most commonly experienced in urban environments, where tall buildings (urban canyons) can inhibit a receiver's direct LOS with a satellite, but can also degrade positioning accuracy for mobile receivers (Schmidt & Phillips, 2010). Instead of the direct signal, the receiver sees the reflected (indirect) signal, unaware that the signal will arrive potentially degraded and with a longer transit time. Further complicating the situation is that a single signal being reflected by multiple buildings may arrive at the receiver more than once. The result is poor positioning accuracy, extended fixing times, and the general unreliability of the service.

Positioning errors can also be caused by faults or problems with the receiver. In terms of the antenna/receiver design, hardware biases, electromagnetic interference, and significant distance between signal reception and signal processing can result in delayed signal processing, causing small interruptions in the TOF calculation.

The collection of GNSS errors can be classified into two categories: common-mode errors that would affect nearby receivers similarly (satellite biases, atmospheric, ephemeris, etc.) and non-common-mode errors which would affect the nearby receivers differently (receiver bias, noise, multipath, etc.) (Rahman, Silva, Jiang & Farrell, 2022). In order to make GPS as accurate as possible, it must account for as many of these sources of positioning errors as possible- regardless of their relative significance. Earth's imperfect rotation is an example of the more minor sources of errors that more advanced receivers can consider to achieve more accurate positioning (Kumar & Moore, 2002).

2.7 Fixing Time

The TTFF is the time that a GNSS receiver requires from being powered-on to calculating its first three-dimensional PNT solution (Cozzens, 2022b; Fu & Lv, 2021; Linty, 2014). This constitutes time for the minimum position estimate requirements to be met to solve the positioning equation: four satellite signals to be received and processed, as well as the receiver's computation of a position estimate (Lachapelle & Rao, 2012). Unlike positioning, TTFF is not dependent on relative satellite positioning or DOP. Instead, TTFF is dependent on how much information the receiver has on the satellites, system time, and its location when powered on (Lachapelle and Rao, 2012). After the first fix, signals from more than just four satellites will help the receiver close a position estimate to an error ellipse, increasing the accuracy of the position estimate and sometimes quantifying that error ellipse to the user.

TTFF is an important parameter for receiver performance, and is used by manufacturers to indicate equipment capability (Fu & Lv, 2021). Receiver manufacturers thus patent novel architectures and software in order to present industry-leading low TTFF to customers (Siemuri *et al*., 2022). Even though TTFF is the subject of many studies, researchers seldom discuss the quality of those first fixes (Anghileri *et al*., 2008; Cao, 2008; LeVeel, n.d.; Luo, Chen & Richter, 2017).

The absolute minimum data required by a receiver to estimate its position is ephemerides of four satellites, GPS time, and the applicable clock correction. The time required of a device to estimate its position is thus influenced by two major constraints: signal acquisition and data download. However, the speed that a receiver can retrieve the necessary data and compute a first fix is dependent on (but not limited to) the following (Linty, 2014):

a. Satellite availability;
b. The navigation data message;
c. Receiver architecture; and
d. Receiver state.

2.7.1 Satellite Availability

Foremost, TTFF depends on the availability of GPS satellites for without which, a position cannot be estimated (Langley, 2015). Required is a direct LOS between the antenna and at least four satellites; obstructions between the antenna and satellites result in interruptions in the data download, forcing the receiver to either restart or listen to the same frames again, extending the TTFF. Users may experience extended or even indefinite time for an initial fix in conditions or environments of poor GPS coverage, such as in canyons or urban settings. The antenna's FOV should be maximized and obstructions minimized, enabling continuous downloads of all required data.

An increased number of visible satellites can help reduce TTFF. Better satellite geometry would also stem from additional satellites, improving positioning accuracy (Measurement Systems Limited, 2023). The availability of augmentation systems, signals of opportunity, and assist data can also help in reducing TTFF (Linty, 2014).

2.7.2 The Navigation Data Message

If satellites are available and can be acquired, the TTFF also depends on the structure of the GPS navigation data message – foremost the time required to download the necessary ephemeris, system time, and clock data, which can be considered the most significant factor in TTFF (Anghileri, 2013, Langley, 2015, Yang, 2018).

Upon being re-initialized, if no location information was saved in NVM, is invalid, or was lost, the receiver will search for satellites from a default location, such as its manufacturer's location (Grewal, 2020). If the receiver has been relocated a substantial distance from its last known position, like on the other side of the planet, it will be looking for satellites in the sky that may be on the other side of the Earth, which can lead to an extended time to fix while the receiver categorizes the available satellites (Grewal, 2020). The receiver searches the sky for all known GPS satellites PRN codes (between 24 and 32, depending on the date in question) - a process which could take several minutes since most will not be visible. The more channels a receiver has, the quicker it can conduct this search and calculate a position estimate as the receiver works the satellite PRN codes in parallel. In order to reduce the TTFF, the almanac was introduced to provide devices with a manner of foreseeing which satellites should be within their general FOV given their position on Earth and the time. With a valid almanac, the receiver can disregard satellites it knows to be out of view for its current position and time, focusing processing power on the satellites overhead. The user can save time by uploading the receiver's approximate location and using the existing almanac to acquire the available satellites, or by manually clearing the existing almanac data and allowing the receiver to start a fresh satellite acquisition process (Grewal, 2020). Thus the receiver would not have to search for satellites signals from satellites not within its current FOV, shortening the TTFF. This almanac data would be continuously available to reduce future fixing times, although only valid for short periods of time.

Unfortunately for the user, there is nothing they can do to reduce this time – it is thus an inherent wait time for any positioning through that particular GNSS.

2.7.3 Receiver Architecture

The receiver's architecture including its design, signal processing, antenna type, firmware, and number of channels all help determine TTFF. However, the signal acquisition strategy, signal processing techniques, and proprietary algorithms of receivers are generally not available in literature, protected by manufacturer patents and thus limited in research potential for receiver enhancement (Linty, 2014).

A receiver's rapid acquisition of satellites is a key factor in minimizing TTFF. A receiver with more channels can acquire more satellites simultaneously, and download and process the necessary navigation data. If the receiver has loaded almanac data saved in NVM that outlines what satellites are directly overhead as it is powered on, it can lock onto those satellites immediately and listen for those satellites' ephemerides. A receiver's ability to listen to multiple frequencies of one constellation or even different constellations can also reduce TTFF (Cao, 2008).

Since the submarine's GPS receiver is designed to be integrated with an INS, it can be designed differently than a standalone receiver. One design difference is the narrower bandwidth for carrier and code tracking loops (Schmidt & Phillips, 2010). The lower bandwidth improves signal tracking in noisy environments, including interference/jamming situations. The improved tracking makes the receiver less susceptible to jamming by a factor of three-to-four (Schmidt & Phillips, 2010).

However changes in hardware to reduce TTFF often come with drawbacks. For example, while additional channels enable the tracking of more satellites, the processing power and electrical draw also increase. Another example is an increase to the receiver's sensitivity, which can reduce TTFF, improve tracking, and enhance receiver performance in GNSS-challenging environments (Noureldin, Karamat & Georgy, 2012); however, higher receiver sensitivity consequently makes the receiver more vulnerable to jamming (NXP, 2009). In order to maintain low TTFF without compromising receiver sensitivity, the manufacturer can incorporate a low noise amplifier in proximity to the antenna (NXP, 2009). Complex trade-offs like this are continuously improving receiver performance particularly for high-precision high-accuracy positioning applications (Linty, 2014).

2.7.4 Receiver State

Lastly, the time required to download the necessary navigation data is a major factor in TTFF, which is determined by its state (Couronneau, 2011; Hubert, 2022). GPS receivers power-up in one of three states - hot, warm, or cold – where what data is valid varies in each case. Factors affecting the start-up state include the ephemeris and almanac data being valid (no more than four hours and 180 days respectively), time since last fix (no more than three days), and distance from last fix (no more than 100 kilometers) (Measurement Systems Limited, 2023). A cold start, the worst case scenario, requires the greatest amount of time for a first position fix since it lacks the most information.

2.7.4.1 Cold/Factory

A receiver will be in the cold mode when it has no information on the satellites, its position, or the time (Hess, 2015). Usually this is the case when the receiver is powered up for the first time, but can also occur if power is lost and there is no memory to store previous data. If this is the case, the receiver starts by acquiring any visible satellite(s) through a full-sky search and downloading the complete almanac which is repeatedly transmitted by all GPS satellites every 12.5 minutes (Anghileri *et al*., 2008). Outputting a first position estimate without any assist data or augmentation services could end up taking up to 15 minutes (Brown, 1996; Griesel, 2006; Measurement Systems Limited, 2023).

Before the full almanac download is complete, a receiver can output rough positioning information within the first couple minutes; however, that first fix is relatively inaccurate (Anghileri *et al*., 2008; Lachapelle & Rao, 2012). At a minimum, required data is ephemeris and clock correction data, and the GPS time reference (Anghileri *et al*., 2008).

Other than a manual reset that erases all stored location and satellite information, the receiver can be defaulted into a cold start if it loses power (and thus loses time information), its loses satellite information, it is inactive for a period of time, or it travels in excess of 100 kilometers without a position fix (Measurement Systems Limited, 2023).

Receiver manufacturers are continuously seeking strategies to reduce fixing times, particularly for cold starts which demand the most time before outputting a position estimate. One strategy is the storing of almanac data in NVM so it is available upon receiver power-up (as long as it is still valid). Research like that by Lachapelle & Rao (2012) announces only between 34 and 36 seconds are required for a cold start, likely from the availability of stored almanac data or assist data.

2.7.4.2 Warm/Normal

A warm start is conducted if the receiver has conducted a recent successful fix and thus knows its approximate position (Hess, 2015). The receiver should have valid almanac data stored within NVM, the accurate time within 20 seconds, its position known within 100 kilometers (approximate user position), and its velocity within 25 meters per second. As long as a device retains power to keep track of time, its embedded memory retains position information to expedite a first fix.

Upon being re-initialized, the receiver takes into account the last known GPS time (with its own time tracking since the last fix), its last known position, and its stored almanac data to assess which satellites should be within its FOV. The receiver then restricts its satellite search and identification algorithms to only those satellites rather than the entire constellation (Lachapelle & Rao, 2012). The receiver acquires the visible satellites and immediately downloads their ephemeris data before computing a positon fix. With each satellites' ephemeris data transmitted on 30-second loops, typical TTFF is between 30 and 45 seconds, giving time for signal processing (Lachapelle & Rao, 2012; Measurement Systems Limited, 2023).

The device needs to retain power in order to keep track of time. If power is lost, the device has no sense of time and so the stored position and almanac data are useless, forcing the receiver into a cold start. Since GNSS receivers are passive, some receivers update their satellite information even when being "off" or in simulator mode as long as power is available, in order to abbreviate the fixing time when prompted by the user. This can be useful for devices that can move great distances when powered off, in a car for instance, before being needed by the customer again. As long as the device has power and a NVM, the almanac, position, and time can all be kept up to date in the background.

2.7.4.3 Hot/Standby

With a last position fix usually within the last hour along with valid almanac, ephemeris data on visible satellites, time, position, and velocity information, the receiver can rapidly acquire the required satellites and calculate its position without first downloading data from the navigation message (Anghileri *et al*., 2008). Typical TTFF is under a minute, with an average time of 22 seconds, but can be as quick as only a few seconds (Measurement Systems Limited, 2023).

A receiver in the hot/standby mode will retain satellite information and provide a continuous position estimate. There is no need to acquire and re-connect to the satellites for each subsequent fix. Receivers in the hot state can be referred to being in standby as it can continuously provide real-time positioning data. The only new information required are the new ephemerides from satellites appearing at the horizon. With the satellites in continuous motion around the Earth, satellites in view will eventually fall below the horizon but will be replaced by new satellites. Channels beyond the four required for a three-dimensional fix can monitor the horizon and download new ephemerides before the satellite is actually needed.

A receiver in the hot mode can revert to the warm or cold state if certain conditions are not met, particularly if the receiver has no visible satellites for a particular distance or period of time. If this is the case, a reversion in receiver state, from hot to warm or even to cold will always result in a slower TTFF as new satellite data is required. The first position estimate after the receiver is initialized after this change in conditions will impact the TTFF.

2.7.4.4 Receiver State Summary

The conditions of the three start modes, with data requirements, and typical TTFF are summarized in Table 2. The more data that can be retrieved from NVM (if still valid) or from other available sources results in a reduced TTFF. Due to the abundance of assist data available through terrestrial networks, and frequency of device use, warm starts are more popular than cold starts. Thus research into reducing TTFF mainly focuses on warm start conditions, or the missing data that pushes a device from a hot start into a warm start (Samson, 2011).

Table 2. Summary of the GPS states and associated TTFF (Measurement Systems Limited, 2023).

State	Conditions	Requirements to fix	Typical TTFF
Cold/Factory	>100 km since last fix; or Three days of inactivity; or Missing position, time, or satellite information; or Manually reset	Acquire satellites, download almanac, download ephemerides, GPS Time, Clock Correction range off four satellites	1-15 minutes
Warm/Normal	Valid almanac; Time within 20 sec; Position within 100 km of last fix; Velocity within 25 m/s;	Acquire satellites, download ephemerides, GPS Time, Clock Correction range off four satellites	30-45 seconds
Hot/Standby	Valid almanac, ephemeris, time, position, and velocity	Range off four satellites	Few seconds

It is recommended by receiver manufacturers to initialize a receiver prior to starting an activity, particularly if that activity brings the user to a remote area. The Garmin ZŪMO 396 Owner's Manual (2018) informs customers that "the time required to acquire satellites signals varies based on several factors, including how far you are form the location where you last used your navigation device, whether you have a clear view of the sky, and how long it has been since you last used your navigation device." Thus TTFF is kept to a minimum if the receiver is left on and locked to GPS satellites- although this configuration does consume more power.

In the case of the submarine, a cold start can occur if information is mission or is invalid due to the submarine travelling in excess of 100 kilometers while underwater, resulting in an extended wait time for positioning information. The most likely scenario particularly for conventionally-powered (diesel-electric) submarines on patrol, which surface once every day, is that only the ephemeris data is expired upon their return to the surface (Hess, 2015). Without valid ephemeris data, the submarine's receiver is forced into a warm start, requiring approximately 30-45 for positioning data (Measurement Systems Limited, 2023).

2.7.5 Other Factors

Like other satellite communication systems, the electromagnetic radiation that makes up the satellite signals is subject to attenuation by a number of natural phenomena. Signal attenuation caused by weather, humidity, and atmospherics can degrade GPS signals, leading to data interruptions which force receivers to restart the download and consequently extend the TTFF. Extreme conditions can even impede satellite acquisition altogether, forcing the user to acquire positioning data from other sources/methods.

2.8 Time To First Fix Calculation

The display of a position estimate by a receiver requires a sequence of events, which when accumulated, can represent the device's TTFF (Lachapelle & Rao, 2012). The majority of the TTFF is spent in satellite acquisition and reading of the navigation message, which are influenced by the four previously discussed factors (Anghileri *et al*., 2008). Individual devices' strategies and algorithms will only play a small part in reducing TTFF. The following is a breakdown of the time between receiver initialization and the first position estimate.

Firstly, the time for the receiver to boot/power-up can be defined as T_{boot}, which includes the time to load the signal processing software and application that will ultimately display the PNT information (Anghileri *et al*., 2008; Lachapelle & Rao, 2012). The GPS+GLONASS receiver used in the TTFF tests of Lachapelle and Rao (2012) had a T_{boot} of approximately 250 milliseconds while the multi-GNSS receiver tested in Anghileri *et al*. (2008) saw T_{boot} of approximately 2 seconds.

Once the receiver has booted, the receiver's number of channels determines how many satellite PRN codes its can process simultaneously. The time required to acquire satellites within its FOV, $T_{acquire}$, is a function of the receiver's search space and search strategy (Anghileri *et al*., 2008). Original GPS receivers with only four channels were slow at acquiring satellites, potentially taking up to 30 minutes for this as they scanned across all possible Doppler-shifted frequencies of all orbiting GPS satellites in an attempt to lock onto any of them (Wells, 1987). Modern receivers that can store almanac and last fix data in NVM can significantly reduce their search space since they have a general understanding of what satellite should be within view. Additionally, their use of many more channels and new signal processing techniques have shaved GPOS fixing times down significantly, usually to within a minute of being initialized.[9] Based on its last known position and the almanac, the receiver gauges the range and range rate of the satellites to distinguish the GPS satellite signals from background noise and ultimately achieve lock-on (Schmidt & Phillips, 2010). Tests by Lachapelle & Rao (2012) saw $T_{acquire}$ last between two and eight seconds. In addition to one of the major factors in TTFF, signal acquisition is also the primary power draw in GPS positioning, so

[9] A receiver not displaying a position estimate after 20 minutes indicates that something is wrong and the receiver should be reset and/or moved to a different location for a better FOV of the satellites overhead.

reducing the number of times the receiver initially acquires satellites can save power. Some manufacturers and users opt to keep their receiver's TTFF low by either staying powered-on or remaining in a standby mode, where it maintains its connection to the in-view satellites but does not necessarily update its EP.

After satellites are acquired, the channels conduct bit-synchronization ($T_{bit\text{-}synchronization}$) lasting approximately 800 milliseconds (Lachapelle & Rao, 2012). Note that each channel that holds a satellite conducts satellite acquisition and bit-synchronization simultaneously.

Once a channel is synchronized, it can begin downloading navigation data from the satellite – a process that takes the largest percentage of the TTFF timeline (Lachapelle & Rao, 2012). The download of almanac data, if required, begins from the first satellite acquired. The time required for complete data download, $T_{nav\ data}$, is dependent on the validity of navigation data stored in NVM, the navigation data message structure, and the receiver's state. Once the required data is downloaded, the information needed to calculate pseudoranges is retrieved, a process whose duration is termed the read time (Anghileri *et al*., 2008). Since a receiver can catch the satellite broadcast and start downloading navigation data at any point within its broadcast cycle, the reading start point can be considered a uniformly distributed random variable distributed over the 30-second frame (Anghileri *et al*., 2008).

Once all the required navigation data is downloaded, the receiver can measure the incoming timing signals and compute its position relative to the satellites ($T_{compute}$). The time for the positioning computation is mostly dependent on the algorithms and embedded processes (Anghileri *et al*., 2008). $T_{compute}$ can be longer for a receiver in the cold state since it has no prior knowledge of the user's position (Anghileri *et al*., 2008). For a warm start, since the user's approximate location is known, $T_{compute}$ will be smaller. In a hot start scenario, $T_{compute}$ can be considered negligible (Anghileri *et al*., 2008).

The components of TTFF are all affected by signal strength, where good signal strength results in low TTFF (Noureldin, Karamat & Georgy, 2012). Altogether, the time required for any device to calculate an estimated position from a cold or warm start can be expressed as:

$$TTFF = T_{boot} + T_{acquire} + T_{bit-synchronization} + T_{nav\ data} + T_{compute} \quad (4)$$

Although a fix can ultimately be displayed at this point, the receiver can refine its solution as more satellites are acquired and known errors reduced.

Table 3 shows that TTFF is mostly influenced by the satellite acquisition time ($T_{acquire}$) and the download of satellite data ($T_{nav\ data}$) (Lachapelle & Rao, 2012;

Samson, 2011). However, these times are design parameters of the GNSS itself and are out of control of the user. They cannot be manipulated or adjusted in order to reduce TTFF. Thus receiver manufacturers seek alternative methods to keep TTFF at a minimum. For example, receivers can be programmed to only download the data that is absolutely required for a quick fix (less than the 30-second frame), even if that results in a less accurate position estimate (Amghileri, 2013). Other alternative receiver architectures and signal processing techniques are protected by patents and thus not available in open literature for research and analysis (Linty, 2014).

Table 3. Breakdown of TTFF parameters (Fu & Lv, 2021; Lachapelle & Rao, 2012).

TTFF Parameter	Process	Approximate Percentage of the TTFF
T_{boot}	Receiver boot time	1 %
$T_{acquire}$	Satellite Acquisition time	18 %
$T_{bit\text{-}synchronization}$	Bit-synchronization time	2 %
$T_{nav\ data}$	Navigation data download time	73 %
$T_{compute}$	Position computation time	6 %

In terms of a submarine, the TTFF plays a role in the wait time between antenna exposure and processing of accurate positioning data. The integrated GPS/INS provides some assistance in reducing TTFF and convergence time. The INS's EP and velocity information is fed to the receiver (known as acquisition aiding in Figure 10) to help reduce the receiver's search time in re-acquiring signals (Schmidt & Phillips, 2010). The use of PPS receivers also allows direct P(Y) code acquisition, while the reduced carrier and code tracking loop bandwidth help acquire and track satellite signals particularly when maneuvering and experiencing high noise (Schmidt & Phillips, 2010). While the acquisition aiding as part of the loosely-coupled GPS/INS architecture helps reduce the submarine receiver's search time (Schmidt & Phillips, 2010), the time saved compared to the entire $T_{acquire}$ and $T_{nav\ data}$ is relatively small[10]. The reduced bandwidth will also reduce the probability that signals are lost and the satellite must be re-acquired compared to a standalone receiver.

While receiver manufacturers can commit R&D into delivering the lowest TTFF among their competition, it is important to note that there is no accuracy limit associated with the receiver's first fix presented to the user. A quick fix,

[10] The amount of time saved in acquisition is also dependent on the accuracy of the INS position and velocity estimates (Schmidt & Phillips, 2010), which degrade with time underwater as the POE grows.

although inaccurate, is lucrative for manufacturers looking to advertise low TTFF to attract customers. Alternatively, some receivers will withhold position fix information from the user until it meets an accuracy threshold, preventing inaccurate positioning information from any navigation applications. While a receiver may feature a low TTFF, it is important to note the quality of that fix so the positioning information does not jeopardize safe navigation.

2.9 Convergence Time and Accuracy Threshold

In most GNSS research, GNSS performance is assessed through evaluating long-term positioning accuracy over the course of hours or even days (Lim, Yoon, Cho, Yoo, & Park, 2019). Not many studies focus on the short-term accuracy since the majority of GNSS users either have their receivers powered-on all the time or can afford to wait for the positioning solution to converge for accurate data. With a submarine's GNSS connectivity being intermittent, submariners are concerned with short-term GNSS performance and in particular getting accurate positioning data as quickly as possible.

It is not uncommon for a receiver's first position estimate to be relatively inaccurate. The inaccuracy of the first fixes following four GPS warm starts relative to the receiver's accuracy in the subsequent minutes is seen in Figure 11. Beyond the TTFF is the time required for the positioning solution to converge to its maximum accuracy, which can take many minutes, hours, or even days (Rychlicki, Kasprzyk & Rosinski, 2020). Following $T_{compute}$ to achieve the first fix, the receiver acquires additional satellites, continues to download satellite (and augmentation) data, and applies corrections for noise and known errors, such as ionospheric delay, in an effort to improve positioning performance (Lachapelle & Rao, 2012). Positioning accuracy then plateaus for the duration that the receiver is operating, as long as satellites remain available and within the antenna's LOS. Only small variations in accuracy appear in the long-term, mostly caused by changes in satellite geometry (DOP).

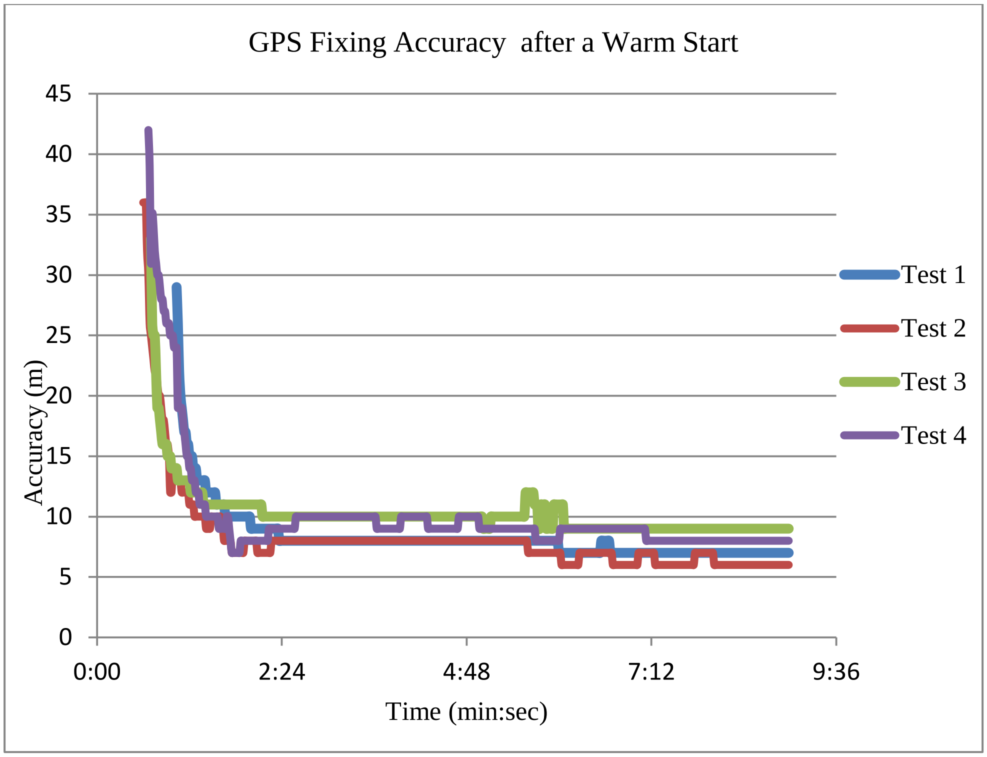

Figure 8. GPS solution accuracy following warm starts.

Following the accumulation of drift and error while underwater, the INS periodically requires accurate GNSS measurements to correct its position, reducing the size of the POE to its smallest size (Grewal, 2020). Complex navigation systems, like those on the submarine, implement an accuracy threshold to prevent the system (and its sub-systems) from accepting inaccurate GPS positioning information. The accuracy threshold of an integrated GPS/INS (not necessarily onboard a submarine) would depend on a number of factors, including its requirements, sensors, processing, calibration, alignment, errors, algorithms, and maintenance.[11] In its loosely-coupled architecture, the Kalman filter also monitors the sensor (GPS in this case) output for incorrect/inaccurate positioning data that could negatively impact the INS (Marvel, 1998). When the GPS data is valid, it also helps estimate INS sensor errors[12] which can help reduce the POE growth rate

[11] A submarine's GPS/INS accuracy threshold is not published, since it along with the INS's performance when out of GPS connectivity will determine how long and far the submarine can safely navigation underwater without having to surface for a positioning update.

[12] Alignment error and computational error cannot be eliminated with the periodic GPS fix. These errors are constant and remain within predictable limits over time (Rogobete, 2018).

between GPS position updates (Cox & Wilfong, 2012; Rogobete, 2018). When the submarine dives again and GNSS signals are inevitably lost, the INS is then in the best possible state to sustain safe navigation with a low POE growth rate.

Following the first fix (TTFF), the positioning solution converges and will have its first "good fix" which meets the accuracy threshold- a pre-determined value relative to the receiver's average solution (Marvel, 1998). Prior to that point, the positioning data should be ignored. Only once the data refines to the accuracy threshold can the user have confidence that the GPS data is accurate and will update the navigation systems accordingly, ensuring safe navigation amongst other purposes. However, the time from receiver initialization and the first fix that meets the accuracy threshold, and convergence (maximum possible accuracy) time can be two different values. Depending on the INS performance, the accuracy threshold is the positioning accuracy that will be accepted for an INS update, which is not necessarily the GPS receiver's minimum accuracy.

This paper thus defines the term Time To First Good Fix (TTFGF) - the interval between receiver initialization and the navigation solution meeting the user's accuracy threshold. For the submarine, the accuracy threshold is that of the integrated INS, which will only accept positioning data that is known to be valid and a true representation of the submarine's location. In the submarine's loosely-coupled GPS/INS architecture, the responsibility of ensuring only valid, good fixes are passed onto the INS falls to the receiver's Kalman filter.[13] Mathematically, TTFGF is equal to TTFF plus the time required for the solution to converge and ultimately meet the accuracy threshold. This accuracy threshold can be set differently depending on the use of the positioning information. Only positioning data collected after the TTFGF will be used in navigation.

$$TTFGF = TTFF + T_{converge\ to\ Accuracy\ Threshold} \tag{5}$$

While TTFF dominates in receiver comparison and multi-GNSS performance studies, convergence or the quality of that first fix is rarely analyzed or discussed (Anghileri *et al*., 2008; Cao, 2008; LeVeel, n.d.; Luo, Chen & Richter, 2017; Rahman, 2022). While TTFF parameters are an important factor in receiver procurement, it is the solution convergence that follows the TTFF that dictates how useful the receiver will be in the short-term. For a submarine, the TTFF is a more minor role compared to the convergence rate while waiting for a fix that will meet the accuracy threshold. Quick but inaccurate GPS information inputted into the integrated GPS/INS as either initial conditions or as a position update would lead to a faster accumulation of errors than the INS is inherently subjected to, which

[13] With the aid of the INS to determine the validity of GPS measurements, tightly-coupled architectures are superior at rejecting poor GPS positioning data.

could inhibit the submarine’s underwater navigation (Boguspayev *et al*., 2023). Inaccurate positioning data provided to the submarine’s weapons and communication systems would similarly degrade their performance.

The GPS positioning accuracy of just Test 1 in Figure 11 is replicated in Figure 12. While the receiver had a TTFF of 1 minute and 2 seconds, the positioning accuracy of that first fix has a horizontal positioning error of 29 meters. For a navigation system with an accuracy threshold of 8 meters, that first fix would not be acceptable. The receiver required an additional 1 minute and 20 seconds to achieve the 8-meter accuracy threshold. Thus the TTFGF in this case – the wait time for the first “good fix” that could be used for safe navigation and relayed to the integrated GPS/INS - was 2 minutes and 24 seconds, more than twice the TTFF value. Still, the receiver requires an additional 4 minutes and 12 seconds to fully converge on its maximum accuracy of 7 meters. For the submarine, positioning data could be accepted at the 2:24 minute mark since it meets the INS accuracy threshold.

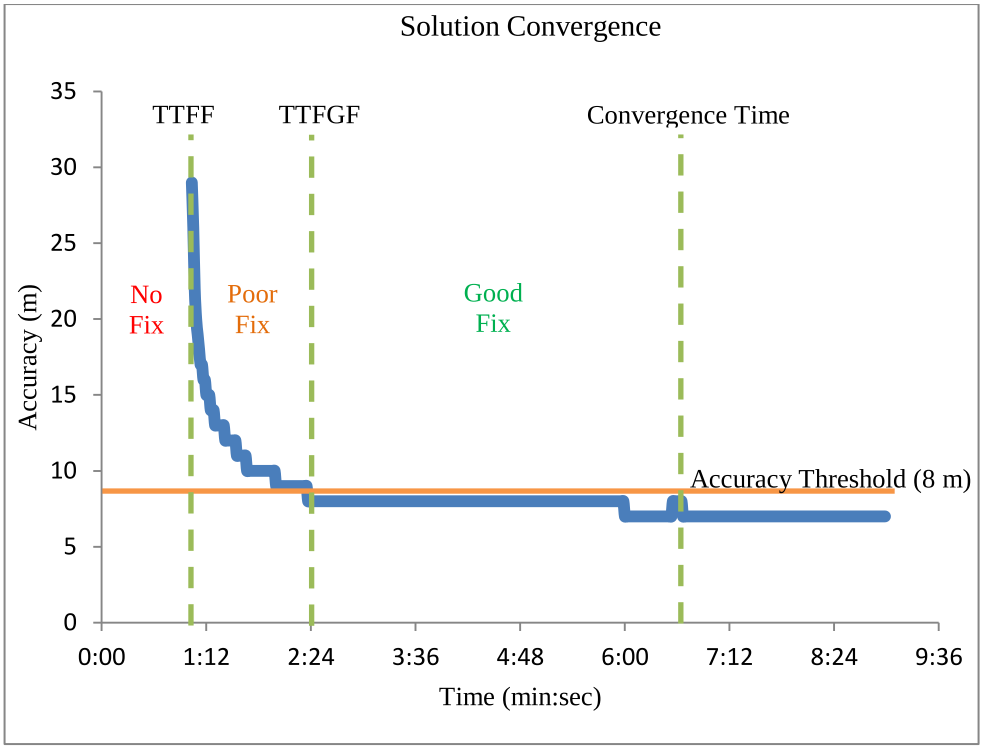

Figure 9. A GPS solution converging following a warm start, with distinct first fix, first "good" fix that meets an accuracy threshold, and convergence times illustrated.

While Lachapelle & Rao (2012) outline that TTFF is dependent on four major factors previously discussed (satellite availability, data message structure, receiver architecture, and receiver state), $T_{\mathit{refine\ to\ Accuracy\ Threshold}}$ is dependent on traditional positioning accuracy factors, namely satellite availability and DOP. Rychlicki, Kasprzyk & Rosinski (2020) notes a near-linear relationship between HDOP and positioning accuracy, meaning that variations in HDOP are a representation of how positioning accuracy will change. This means that more satellites with good geometry will help expedite the convergence rate and lower the TTFGF. Good signal strength as well as the availability of corrected GPS techniques can further refine positioning accuracy quickly for a low TTFGF (Anghileri *et al*., 2008). Surrounding mountains and multi-path effects from infrastructure can limit which satellites can be seen and thus used for positioning, which can prolong the convergence of the positioning solution (Hess, 2015). Altogether, a low TTFGF prioritizes (in order) rapid satellite acquisition, retrieval of the necessary navigation data, and quick refinement of the position estimate through good satellite availability, good satellite geometry, and exploitation of any corrected GPS methods (Cao, 2008; Rychlicki, Kasprzyk & Rosinski, 2020; Siemuri *et al*., 2022). This is summarized in an equation for TTFGF in Figure 13.

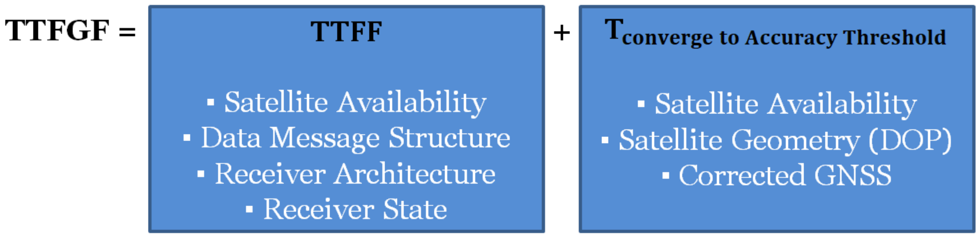

Figure 10. The author's TTFGF equation with the GNSS factors involved in each variable.

Aside from the refinement of a navigation solution following a cold or warm start, an integrated GPS/INS with an accuracy threshold can also isolate GNSS failures from natural sources of error (ionospheric for example) as well as manipulated/spoofed signals. Irregular solar activity can result in an unstable ionosphere that makes satellite communications unusable for certain periods of time (Marvel, 1998). Like with GPS signal spoofing, an integrated GPS/INS system could help identify erroneous GNSS solutions and isolate them from navigation computations (Marvel, 1998). Like when GPS solutions are ignored until they meet the threshold accuracy, short periods of GPS data can be discarded by a loosely-coupled architecture if they have accumulated too much error and exceed the threshold accuracy, until the accuracy recovers (Marvel, 1998). In a tightly-coupled architecture, small errors that exceed the threshold accuracy may go unnoticed and uncorrected (Marvel, 1998).

2.10 Corrected GPS

The method of determining a position from GNSS satellites alone is both slow and error-prone as previously discussed. Inaccuracies can occur from non-ideal satellite geometry and the limited number of satellites and signal frequencies, as well as atmospherics and interference. Positioning with only raw GNSS signals, which provides results adequate for general applications and commercial LBS, can be interpreted as “uncorrected” GPS (Schrock, 2021). However, uncorrected GPS is also synonymous with the longest fixing time, as all data is being broadcast by satellites at low power.

The concept of “corrected GPS” (or corrected GNSS) is based on the availability of non-GNSS systems, including Ground-Based Augmentation Systems (GBAS) or Space-Based Augmentation Systems (SBAS), that can augment positioning performance (Wang & Walter, 2023). Correction signals or signals of opportunity can be used to mitigate certain sources of error to either refine positioning accuracy or reduce fixing time, or both. Implementing these concepts can open the GPS market up to applications that demand for more rapid and accurate positioning information.

2.10.1 Assist Data

The most abundant GBAS correction method is the use of assist data (termed Assisted GPS or A-GPS) available from nearby infrastructure. Mobile devices are now designed to exploit any available assist data from crowd-sourced WIFI/hotspot networks and/or cellular towers (Figure 2) to generate a positioning solution at a fraction of the time it takes to acquire satellites and download their broadcast from space. In general, the more abundant and accurate the assist data, the lower the device positioning start-up time (Noureldin, Karamat & Georgy, 2012). If a device requires fresh satellite data due to inactivity, the download of ephemeris and clock data is accelerated by their availability on nearby A-GPS servers, which is made available to mobile devices at a quicker rate than if it were downloaded from the satellites (Cozzens, 2022a; Garmin, n.d.c; Noureldin, Karamat & Georgy, 2012). This is a handy feature for times when the device cannot connect to four satellites, such as the user being indoors or in urban canyons, where direct LOS with satellites is limited.

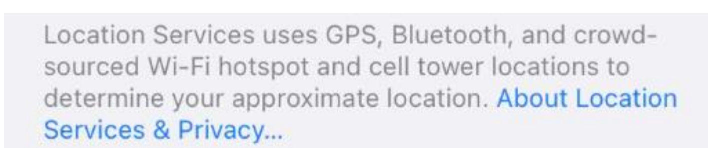

Figure 11. Screenshot of an Apple iPhone's location services, outlining the array of external resources used in determining the user's location.

The connection of cellular and WIFI networks to GPS enables continuous download and subsequent re-broadcast of all relevant GPS data to an unlimited number of users. With GPS data readily available, a compatible device can get a GPS lock almost immediately. The download of assist data from these sources can be automatic by a receiver, helping a device reduce the TTFF and augment its positioning performance at no additional cost to the user. Since assist data is broadcast over the cellular tower's voice network, a receiver does not need a subscription/data plan in order to retrieve A-GPS information[14] (Garmin, n.d.c).

The data packets broadcast via network link is of higher power than the GPS satellites, and is thus faster (and usually more available) than the satellite link (Shokouh, 2013; Zekavat & Buehrer, 2012). Along with the faster broadcast of ephemerides instead of waiting the minimum 30 seconds for ephemeris downloads directly from the satellites, A-GPS shortens mobile devices' TTFF from minutes down to a matter of seconds upon being connected (Shokouh, 2013; Zekavat & Buehrer, 2012).

Although A-GPS positioning may be somewhat less accurate, mobile users save power from quicker data downloads from terrestrial infrastructure compared to weak signals from satellites. By accessing GPS data over a network, as well as increasing the sensitivity to GPS signals, mobile devices are experiencing TTFF of just one second for cold starts (Grewal, 2020). However, the receiver requires a constant link with the network for synchronization and downloads of the required data (Lachapelle & Rao, 2012).

With the benefits of A-GPS touching all aspects of navigation but also public safety, users cannot manually disable A-GPS and the U.S. Federal Communications Commission (FCC) now mandates that all cellular phones sold in the U.S. must be equipped for A-GPS augmentation for semi-precise positioning in less time (Schrock, 2021). However, the benefits of A-GPS are limited by both the cellular carrier's infrastructure power (range) and location, in which being low to the ground results in additional multipath error particularly in urban environments.

2.10.2 Multi-Band GNSS

Multi-band (or multi-frequency) receivers enable increased positioning accuracy by simultaneously measuring multiple signal frequencies from the same satellite and taking the average position estimate between the different frequencies. In addition to being slow to produce a first fix, the first GPS receivers were single-

[14] Data is required though for the phone/tablet's map on which the fix is placed, vice the positioning service. Dedicated navigation devices will often come with many countries' maps already installed, reducing the customer's need to connect to a data source prior to using the device for navigation.

frequency, resulting in poor positioning performance compared to today's standards. As new signal frequencies were introduced, receiver software was amended to operate not only on multiple frequencies but also multiple constellations, which introduced to the concept of multi-GNSS receivers (Cao, 2008). Since atmospheric error is dependent on signal frequency, a receiver's use of multiple frequencies reduces ionospheric and atmospheric errors amongst other discrepancies in real-time (Luo, Chen & Richter, 2017; Keller, 2008; Rahman, 2022). Most common are dual-frequency receivers, which are available to both civilian and military users to increase accuracy without excessive cost increases compared to single-frequency receivers.

In addition to improving positioning accuracy, multi-band receivers can also reduce the effects of multipath error (Luo, Chen & Richter, 2017) and reduce the time it takes for a receiver to calculate a position estimate (Cao, 2008). Multi-band operations also augment system security by complicating the task of interfering with or jamming the satellite signals (Luo, Chen & Richter, 2017).

2.10.3 Differential GPS

Differential GPS (DGPS) is a manner to increase positioning performance through real-time error correction free-of-charge. Reference stations with both known location and GNSS error information broadcast correction data (particularly atmospheric errors) to refine the local GPS coverage out to approximately 20 kilometers and beyond, as seen in Figure 3 (Fisheries and Oceans Canada, 2000**,** Payne, 2010; Yan, 2023).[15] DGPS coverage can provide users with positioning accuracy between 2 and 10 meters, with some users even reporting sub-meter positioning accuracy (Fisheries and Oceans Canada, 2000; Marvel, 1998; Yan, 2023). In their research on Unmanned Surface Vehicles, Cahyadi, Asfihani, Madiyanto, and Erfianti (2022) showed that differential GNSS consistently out-performed independent GNSS, despite surface ships facing positioning accuracy concerns due to the forces of ocean currents, waves, and wind.

[15] DGPS broadcasts do not contain positioning or timing information, nor do the reference stations broadcast assist data which could help reduce TTFF.

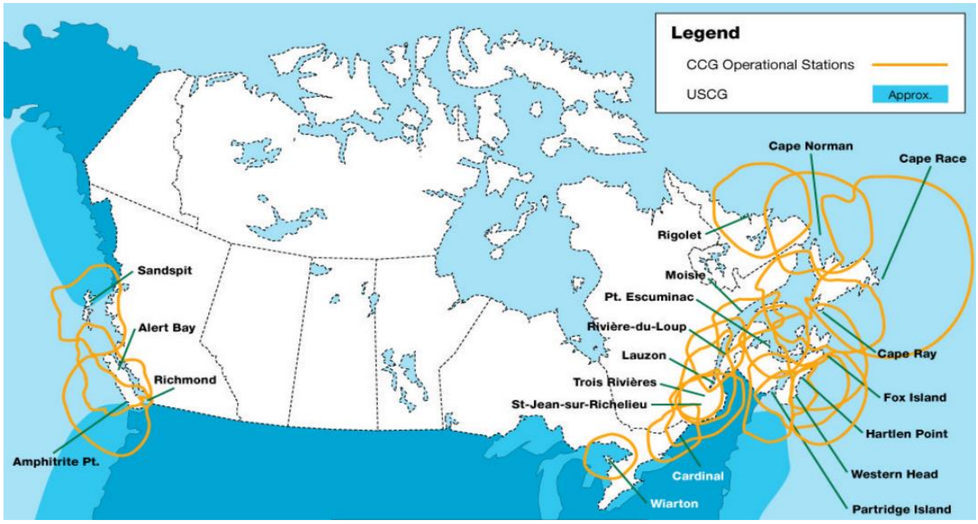

Figure 12. DGPS coverage map for Canada (Fisheries and Oceans Canada, 2000).

However, a user's reliance on DGPS to improve positioning performance is limited by the number of reference stations and the broadcast range. The differential data decreases in validity with distance between the reference station and the user due to two main reasons: different ionospheric and atmospheric errors experienced between the reference station and the user's location, as well as potentially different satellites being used between the two locations. Thus, an extended range of DGPS broadcasts to specifically reach oceanic vessels is undesirable.

2.10.4 Space-Based Augmentation

SBAS is comprised of non-GNSS satellites that transmit range error corrections and/or auxiliary data to improve GNSS positioning and reliability – similar to DGPS but from satellites instead of ground infrastructure. While SBAS can benefit all aspects of a user's GNSS positioning, it is the vertical component (altitude determination) that is the prime benefactor. Thus the most widely deployed SBAS is the Wide Area Differential GPS (WADGPS) whose primary focus is aircraft safety (Kim *et al*., 2016).

Currently operational SBAS (in order of their development) are the Wide Area Augmentation System (WAAS), Michibiki Satellite Augmentation System (MSAS), European Geostationary Navigation Overlay Service (EGNOS), and GPS-aided GEO-Augmented Navigation (GAGAN) with others dedicated to Chinese, Korean, Russian, African, and Australian/New Zealand airspaces currently under development (EUSPA, 2022b). Other more minor SBAS include those onboard LEO satellites (discussed further in Chapter 6). GPS interoperability has been proven with the augmentation of GPS with WAAS for North American civil aviation.

Like DGPS, ground reference stations continuously analyze satellite signals for ionospheric and tropospheric errors; however instead of broadcasting error correction information from terrestrial infrastructure, the data is relayed to users via dedicated satellites, enabling positioning precisions of less than five meters (Schrock, 2021). WAAS has demonstrated the ability to reduce positioning uncertainty by more than half, reducing positional accuracy to within three meters from ten meters (Zacks Investment Research, 2015).

Like GNSS, SBAS services are free of charge. Some new GNSS receivers connect to SBAS satellites automatically while older devices may require a separate receiver to benefit from SBAS's availability. However, as far as augmentation techniques go, SBAS are expensive and only regional, as seen in Figure 4. Specifically for SBAS satellites in geosynchronous orbits, the user's requirement to have LOS with the satellite limits the system's coverage to users at low- and mid-latitudes.

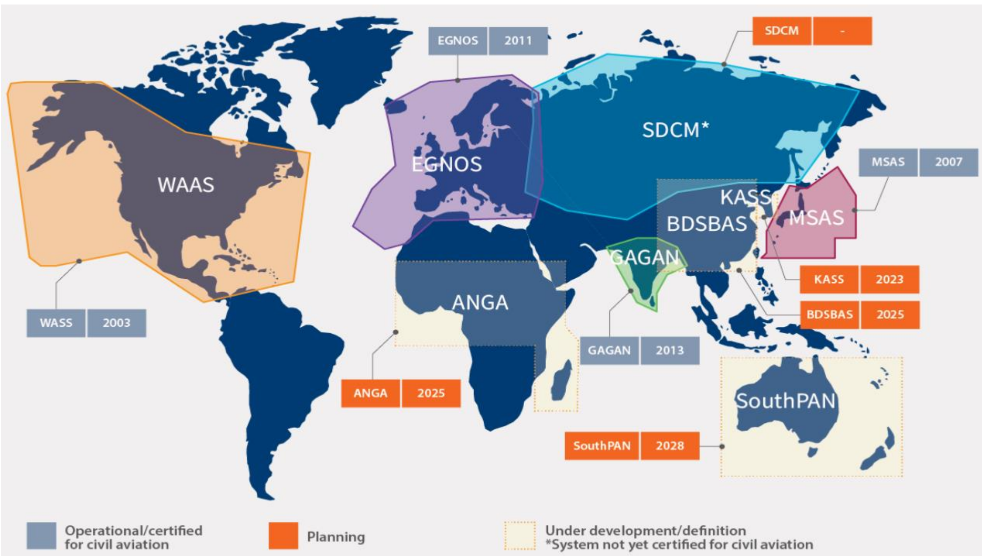

Figure 13. Operational and under development SBAS as of September 2022 (EUSPA, 2022b).

2.10.5 Real-Time Kinematics

Real-time kinematics (RTK) is a method of real-time error correction for precise positioning and reduced TTFF, and is commonly relied upon in low-dynamic scenarios like surveying, robotics, and agriculture. Receiving correction data from a base reference station or reference network in addition to the multiple satellites' carrier waves enables real-time improvements for repeatable centimeter-level positioning accuracy (Cao, 2008; Hadas, Kazmierski & Sosnica, 2019). However, the base stations have limited communication range for their augmentation, and the precision degrades with distance from the base station[16]. A ten kilometer limit can be used as a maximum range guideline for these corrections (Schrock, 2021). Network RTK provides a broadened coverage for these benefits (>10 kilometer limit) and enables centimeter-level accuracies for baselines up to 99 kilometers (Hadas, Kazmierski & Sosnica, 2019). The major drawback of RTK is its higher associated equipment costs with potential subscription fees (Hadas, Kazmierski & Sosnica, 2019; Schrock, 2021).

2.10.6 Precise Point Positioning

PPP is a method of achieving similar centimeter-level accuracy without the dependence on base reference and networks (Hadas, Kazmierski & Sosnica, 2019). Instead, PPP relies on processed reference station common-mode error data to reduce positioning errors (Alkan, 2013; Du, 2022; Rahman, 2022; Schrock, 2021). Special algorithms designed for ambiguity resolution have even helped delivery positioning solutions with millimeter-level accuracy (Hadas, Kazmierski & Sosnica, 2019). PPP has also been proven to help reduce fixing time and help solutions converge more rapidly (Xu *et al*., 2024).

Without the requirement of a dedicated nearby reference station, PPP users are not restricted by range. While some users (seafloor construction or seafloor mapping for example) rely on post-processed historical/logged data for better accuracy, others can opt for real-time PPP (RTPPP) which has faster solutions but costs the user some degree of accuracy[17] and requires a continuous internet connection to receive real-time correction data (Yan, 2023). The PPP concept combined with multi-constellation receivers enables the high precision required for mapping and autonomous navigation amongst other applications but is also associated with expensive equipment. However, in order to achieve the highest levels of positioning accuracy, the user requires long-term continuous observations (Alkan & Ocalan, 2013).

[16] The distance between the base station and the receiver in RTK surveying is often referred to as the baseline (Cao, 2008).
[17] Loss in accuracy in real-time PPP is due to the use of predicted orbit data as opposed to logged data.

A term that appears in PPP-focused studies is convergence time- the time required of a GNSS receiver to reach maximum possible positioning accuracy from initialization (Du *et al.*, 2022; Rychlicki, Kasprzyk & Rosinski, 2020). The major downside of PPP is its association with extended fixing times as the solution converges, usually ranging between 15 and 30 minutes but can be as long as four hours (Du, 2022; Rahman, 2022; Schrock, 2021; Yan & Zhang, 2022). Studies like that by Yan & Zhang (2022) explore the use of models and signals of opportunity to reduce ambiguity and thus improve convergence times, which can be reduced to mere minutes in a method called PPP (Fast) which is augmented by the existing network of cellular and communication ground stations (Schrock, 2021). PPP's ease of application, low-cost, high-precision, and low convergence times make PPP not only an alternative correction technique to traditional DGPS and RTK but their possible replacement (Alkan, 2013; Du *et al*., 2022; Xu *et al*., 2024).

2.10.7 Corrected GPS Summary

Modern commercial receivers can use these methods either individually or in combination to augment their performance, while increasing demand for real-time tracking applications drives R&D into novel strategies. Despite the variety of techniques and methods available today to augment GPS performance, many are impractical to employ in the maritime environment, let alone on a moving platform such as ships and submarines. At the time of this writing, the Royal Canadian Navy (RCN) only relies on GPS satellite signals as its primary external source of positioning and timing data, which is relatively imprecise, error-prone, and slow, with limited DGPS capability for positioning enhancement in coastal waters.

2.11 GPS Fixing for Surfacing Submarines

When a submarine either returns to PD or surfaces after a long time underwater and attempts to regain GPS connectivity, it starts in the warm state. If it almanac data is invalid, it can automatically default into the cold start mode where it requires the download of all navigation data prior to estimating its position (Fu & Lv, 2021; Measurement Systems Limited, 2023). Not being connected to assist servers, submarines must acquire all navigation data form the satellites directly, whose transmissions are low power and consequently slow, requiring at least 30 seconds for a first position fix.

Luckily for the submarine, the open-ocean environment gives the best possible circumstances for satellite acquisition and limited positioning errors. In open waters, there are no sources of signal interference or FOV obstructions, allowing the receiver to receive pseudoranges from any satellites in the sky. Although the surface of the water causes minor fluctuations in signal frequency, there are no real multi-path concerns which on land are responsible for attenuating satellite signals (Hess, 2015)[18]. The submarine's main source of positioning error is ionospheric error. Ionospheric errors are minimized through the use of multi-band receivers, which have also been shown to reduce TTFF and enhance positioning (Cao, 2008).

Although a submarine's GPS receiver retains electrical power throughout its dive underwater, the loss of GPS connectivity upon the antenna entering the water forces a fresh set of satellite acquisitions when the antenna re-emerges. A successful GPS update is only possible if the periscope and its accompanying GPS antenna remain above the surface of the water, with continuous LOS with the GPS satellites. Inaccurate positioning estimates, if connectivity is interrupted by waves for example or the submarine is forced to dive to avoid a collision, are filtered until the positioning information meets the accuracy threshold[19]. Only until the TTFGF has passed and a good position fix obtained can the submarine dive again. In this case, quality is more important than quantity. Since accurate positioning data is more important than quantity of fixes (Hess, 2015), there is no timeline required for the submarine to remain within GNSS connectivity after the accuracy threshold has been met before it can dive again.

Although unlikely, operational requirements may force a submarine deep before a good fix is obtained. Thus it is in the submarine's best interest to acquire a position fix as rapidly as possible (without compromising accuracy) upon breaching the water's surface. In other words, a small TTFGF is required. Unfortunately not all methods and techniques of reducing TTFF and improving positioning accuracy can be applied to submarine positioning due to the unique ocean environment. Furthermore, alternative receiver architectures and signal processing techniques are not an option to reduce TTFGF due to the nature of standardized military receivers and their decryption requirement. The submarine's navigation system is designed for the Trimble receiver and cannot be easily changed for ever-evolving receiver technologies. Alternative manners to reduce fixing time, including other GNSS and augmentation systems, should be explored to ensure submarine stealth as they conduct safe navigation.

2.12 Options to Reduce Submarine TTFGF

This chapter has outlined the factors in both TTFF and TTFGF, along with methods of improving GNSS performance; however, not all can reasonably be implemented to reduce a submarine's TTFF and TTFGF, due to a submarine's

[18] Submarine operations in rivers and fjords will be more impacted by multi-path error (Hess, 2015).

[19] An inaccurate position fix fed into the submarine's navigation system would negatively impact the functionality of a number of other systems, including underwater navigation and weapon guidance.

unique circumstances: mobile, remote operations, intermittent connectivity, and worldwide potential. Changes to the navigation data message, receiver architecture, and receiver state are out of the user's control. The only option to reduce a submarine's TTFGF is through the exploitation of signals of opportunity, which conveniently would be low-cost. As seen in Figure 13, TTFGF is best reduced by improving both TTFF and positioning accuracy. Table 4 outlines the benefits of each signal of opportunity, and further analysis will determine if they could be applied to submarine navigation.

Table 4. Receiver Performance Improvements Summary.

Technique	Improve Positioning Accuracy	Improve TTFF
Assist Data		✓ *
RTK	✓	✓
PPP	✓	✓
SBAS	✓	✓ **
Multi-GNSS	✓	✓

*Limited by network coverage
**Regional coverage only

2.12.1 A-GPS and RTK

A-GPS and RTK are the most widely used GPS correction methods employed to specifically reduce TTFF (Schrock, 2021). However, the necessity of nearby ground infrastructure to receive differential signals negates the potential of employing either one for submarine positioning in remote maritime environments including open-ocean conditions (Yan, 2023). Furthermore, RTK and Network RTK are beset by a maximum range for usable correction data, the utility of which decreases with range. Even if base stations littered the coastlines of the entire planet, the submarine would not be able to access the correction data when sailing in waters beyond their communication range (Schrock, 2021).

2.12.2 PPP

Although PPP has been proven to improve GNSS fixing times and improve positioning accuracy (Xu *et al.*, 2024), there are issues preventing it from being implemented on vessels let alone submarines although a limited number of studies have investigated the use of PPP receivers at sea. One study by Alkan & Ocalan (2013) began with the antenna's static initialization ashore where the solution converged before being mounted on the vessel (Alkan, 2013). Even with a low-cost single-frequency receiver, Alkan & Ocalan (2013) achieved sub-meter level accuracy, although testing was performed in sheltered waters. Yan *et al*. (2023)

confirmed decimeter-level positioning accuracy for kinematic PPP with low-cost multi-GNSS RTK receivers in a remote marine environment, where a vessel would be unstable and moving on all three axes. In the case of a submarine, where the antenna loses connectivity when the submarine dives, the receiver may not be able to achieve the same levels of accuracy since there is no opportunity to conduct static re-initialization ashore every time the submarine surfaces.

Secondly, while the high-precision (centimeter- and decimeter-level) positioning accuracy associated with PPP would be beneficial to the submarine's GPS/INS for long-term submerged navigation (Du, 2022; Rahman, 2022), PPP also requires extended time periods of continuous observations (Alkan, 2013) that would be counter-productive for a submarine trying to remain covert. Since the reduced convergence times and high positioning accuracy capability of PPP demand long-term antenna exposure, PPP is a GNSS commodity that cannot be applied to the submarines.

Lastly, PPP relies on all available GNSS satellites as well as error data from a worldwide network of reference stations. This would require the processing of all GNSS data regardless of political and military relations with their governing bodies, and the internet connection for error data, adding to the requirements for satellite connectivity while at/near the surface.

2.12.3 SBAS

SBAS has significant potential in reducing users' TTFF, and has thus been the focus of prior research. With negligible Doppler shift from geosynchronous satellites, the search space of SBAS satellites is significantly smaller than that for GNSS satellites in MEO, reducing $T_{acquire}$ and thus the TTFF (Samson, 2011). In acquiring SBAS satellites, the User Frequency Offset is determined in a single step compared to the iterative process required when acquiring multiple GNSS satellites, again saving time in the TTFF timeline (Samson, 2011). Lastly, since SBAS satellites are already operational and their services widely available, GNSS users can exploit the time information in the SBAS signal to reduce $T_{nav\ data}$ at no additional cost. Although more of a concern for portable receivers, this strategy of reducing TTFF by reducing both $T_{acquire}$ and $T_{nav\ data}$ would come with the added benefit of reduced power demand since less time is spent on satellite search, signal acquisition, and data download.

However, the studies examining the use of SBAS to reduce TTFF are focused on using SBAS for terrestrial applications and for use with UAVs over land – not in line with submarine operations. The potential use of SBAS signals by submarines to reduce fixing time (and also benefit from increased positioning accuracy) is complicated by two factors: the limited coverage of SBAS over the world's oceans and the system's control. First, the individual SBAS satellites do not provide complete coverage over the entire world's oceans, as seen in Figure 4. Figure 14 shows a more defined coverage map for WAAS, demonstrating the limited coverage beyond the North American coasts into the Pacific and Atlantic Oceans, where RCN submarines train and regularly operate. The sparsely-populated Arctic, where Canada has 162,000 kilometers of coastline (Morrison, 2022), also suffers from a lack of SBAS coverage due to the inherent limitations of geosynchronous satellites. In both open ocean conditions and high latitude submarine operations, the submarine would be forced back to relying on only raw GPS signals, which again is error-prone and slow.

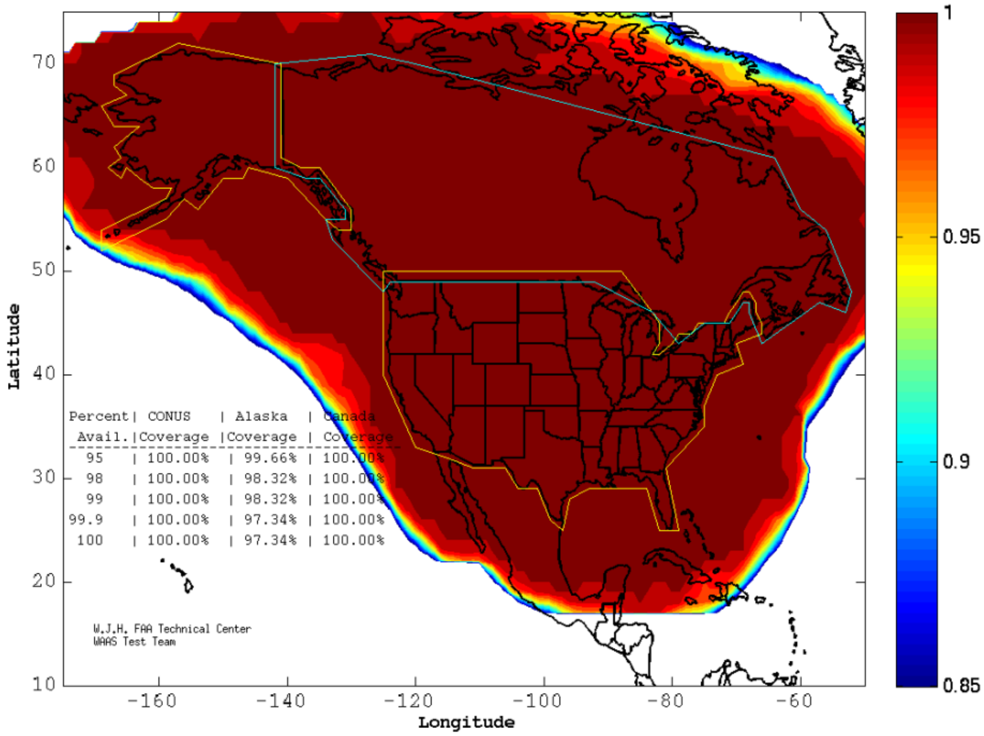

Figure 14. WAAS coverage map of North America (Kim et al., 2016).

The second issue with using SBAS systems for submarines is that although all SBAS across the world are designed to be both compatible[20] and interoperable[21] with GNSS, they are operated and controlled by different governing bodies. They can therefore be tampered with and their availability is not guaranteed, which would limit their use for military operations. WAAS is the only SBAS operated by

[20] No interference between the two systems at the user segment.
[21] Compatible coordinate and timing references.

the U.S., developed for civil aviation in North America and under the day-to-day control of the Federal Aviation Administration. By the same policy rationale applied where GPS is the only reliable system for RCN operations, WAAS would be the only SBAS deemed reliable for GPS augmentation.

2.12.4 Multi-GNSS

Without connectivity to ground infrastructure and requiring greater coverage than that currently provided by SBAS, submarines could exploit the signals of opportunity from other GNSS and integrate them with GPS. Like SBAS, other GNSS have to be both compatible and interoperable with GPS in order for the two systems to work together.

The array of GNSS satellites currently operational means that PNT customers today have more satellites available to them compared to the days of just GPS. Various orbital configurations and data message structures help improve positioning accuracy, reduce interference and error, and reduce both TTFF and TTFGF. If certain signals are blocked, other signals are available for continued coverage. Furthermore, if an entire GNSS fails altogether, other satellites are standing-by for continued PNT solutions. The best part is that these systems are already fully operational and proven, and thus the exploitation of these signals of opportunity for the benefits outlined above is at no additional cost to the user. It is for these reasons that multi-GNSS receivers will be explored in greater depth as the avenue to improve submarine positioning.

2.13 Multi-GNSS Receivers

When GPS was the only operational GNSS, users exclusively relied on its PNT services and augmentations for all their positioning needs (FAA, 2022). As newer GNSS were brought online in the early years of the twenty-first century, users had the option of different GNSS based on their needs or could opt for a multi-GNSS approach. At the time of this writing, GPS consisted of 31 operational satellites, while Galileo had 25 and GLONASS was comprised of 24, making satellites of the three GNSS widely available to users around the world (Trimble, 2023). Once these constellations were all established and proved, the commercial market recognized the benefits and advantages of multi-GNSS receivers for their customers.

Multi-GNSS receivers are commercially available for civilian use (FAA, 2022) and have been the focus of many research studies, particularly as new GNSS mature and their capabilities expand (Hadas, Kazmierski & Sosnica, 2019). These receivers, which are compatible with different GNSS constellations, take advantage of the different signal architectures and satellite constellation structures. The manufacturing of multi-GNSS receivers is facilitated by the similarities amongst all GNSS, including their operating principles and signals (Grewal, 2020). Flexible

GNSS phase processor technology enables receivers to work with multi-frequency and multi-constellation signal processing (Cao, 2008). With the reception of Galileo signals on American devices granted by the FCC in 2018, it is now common for users around the world to see GPS, GLONASS, and Galileo satellites being tracked by their devices. Many mobile and emergency location devices now operate with multi-GNSS capability as a default setting. Although some Garmin handheld devices operate with GPS+GLONASS as their default, they (like others as well) offer users the choice between GNSS combinations as follows:

a. GPS + GLONASS (default Garmin setting);
b. GPS + Galileo; or
c. GPS.

One important consideration in selecting certain GNSS is their orbital configuration. The orbital path along with the number of satellites means that certain systems are more favourable at different latitudes. With limited population at high latitudes, the GPS satellites' 55° inclination results in elevated GDOP values at latitudes above 70° (Hunt, 2020). Another consideration in selecting one or two GNSS over another is the user's environment. Online forums hold discussions of users who express difficulty in good positioning either in mountainous regions or urban centers, where satellites higher in the sky can be of greater benefit.

Just like in the addition of channels to track more satellites at the same time draws more power, so does the use of different constellations (Garmin, n.d.a). Although more of a concern for handheld/portable receivers, users who are concerned with their receiver's battery life can make the following changes to reduce power consumption:

a) Single-GNSS (GPS-only for example) operation: the device acquires signals only from GPS satellites every second[22];
b) Battery Saver Mode: the device will only update its position every three to five seconds, saving power but losing positioning and timing accuracy;
c) Standby Mode: the device will update its navigation data periodically in the background, ensuring it can produce a rapid position estimate if prompted by the user;
d) Simulator Mode: the device does not acquire satellite signals, but simulates signals for planning purposes; and
e) GPS Off: if satellite signals are not required, the device will remain powered-on but the device will not acquire a fix.

In order to have positioning data readily available, receivers will continuously download ephemeris data and clock corrections at different frequencies to save

[22] Preferably the GNSS with the best performance for the user's latitude and environment.

power, so that if prompted by the user, they can provide a position estimate. If power is lost, such as in the event the battery is removed, the device will not be able to track its position and maintain a record of GPS data and time, and will default to a cold start upon initialization.

2.13.1 Benefits

In general, operating a multi-GNSS receiver benefits the user's experience in three main areas: positioning accuracy, system integrity, and TTFF.

2.13.1.1 Positioning Accuracy

Similar to how GPS accuracy was augmented by the introduction of additional signal frequencies, augmenting GPS with other GNSS can improve positioning performance. Compared to operating single-GNSS receivers, research shows increased positioning accuracy with multi-GNSS receivers, prompting manufacturers like Garmin to make multi-GNSS receivers available to their customer base (FAA, 2022). The superior positional accuracy is based on the increased number of visible/available satellites as well as better satellite geometry (FAA, 2022; Noureldin, Karamat & Georgy, 2012; Rychlicki, Kasprzyk & Rosinski, 2020; Trautvetter, 2018). As more global and regional systems are brought online, any GNSS skyplot has more satellites both in view and available today than ever before. Using signals from multiple GNSS results in increased satellite availability for greater positioning integrity, increased system geometric diversity, and reduced TTFF (Cozzens, 2023a). If different GNSS signals are interchangeable at the receiver level and its number of channels suffices, the receiver can use as many satellites as possible for the best possible positioning performance. Different constellations with their unique orbital configurations enable more optimal satellite geometry, resulting in a better DOP and fewer positioning errors, particularly in challenging environments. The different satellite frequencies can also help reduce ionospheric errors, which augment positioning performance (Cozzens, 2023a).

Various research studies have outlined the benefits of multi-GNSS receivers in terms of positioning accuracy. Testing by RX Networks Incorporated for the EU in 2014 demonstrated both improved horizontal and vertical positioning accuracy when using multi-GNSS receivers compared to GPS-alone in both indoor testing and challenging environments like urban canyons, with other results showing a 10% improvement in accuracy (Hadas, Kazmierski & Sosnica, 2019). Results like these are only expected to improve further as more GNSS satellites are launched.

The aviation industry has already implemented multi-GNSS capabilities into their operations, where the Advanced RAIM (ARAIM) uses both GPS L1 and L5 frequencies alongside Galileo E1/E5 for more precise altitude information

(FAA, 2022; Wang & Walter, 2023). Moving forward, Boguspayev *et al.* (2023) suggests that multi-GNSS receivers in tightly-coupled GPS/INS architectures will provide reliable solutions for air and land vehicles that experience intermittent GNSS signal unavailability.

As for handheld devices, even sport watches are incorporating GNSS receivers for maximum positioning performance particularly in high-dynamic athletic situations with continuous arm movements. Wearables like the Fenix 7 and Epix 2 feature an “AutoSelect” option which enables SatIQ (“satellite intelligence”) to select the best possible satellite combination. The option also optimizes power consumption for optimal device performance.

2.13.1.2 System Integrity

The differences in GNSS signal frequencies, data message architecture, and satellite constellation configurations could offer greater positioning redundancy, particularly in GNSS challenging environments. Working with multiple independent GNSS also improves reliability, in the event on GNSS is unavailable.

Jamming and spoofing expose the vulnerabilities of GNSS users, many of whom rely on a single GNSS which can be a single point of failure. With GNSS integral to billions of systems and personal devices around the world and with critical infrastructure having significant dependencies on GNSS, London Economics estimated that a GNSS outage could cost the United Kingdom over $1 billion pounds (approximately $1.7 billion CAD) per day (Lawrence *et al*., 2017). Operating with two or more independent satellite systems together essentially eliminates the chance the user is subjected to PNT degradations due to failure of one system (Loizou, 2020). If one system fails or its signals are degraded, there remains entire other GNSSs capable of three-dimensional positioning. The user is thus no longer dependent on one GNSS owner or its infrastructure. Also, the variety of satellite frequencies used by GNSS receivers augments anti-jamming capabilities, adding to the robustness of PNT services.

2.13.1.3 TTFF

Compared to single-GNSS or single-frequency systems, additional satellites and a variety of signal frequencies also shorten a user’s TTFF in all three start-up modes at no additional cost to the user, as long as the receiver is compatible with both satellite systems (Cozzens, 2023a; Pan, 2014). GNSS simulations before Galileo was fully operational theorized abbreviated TTFF timelines when using more than one GNSS frequency, regardless of the constellation (Cao, 2008). Initial Galileo test satellites working alongside GPS and GLONASS confirmed these timelines (Bowler & Wall, 2014) but many changes to both Galileo and GPS have occurred since that time.

If the different GNSS signals are interchangeable at the receiver level, a multi-GNSS receiver will fix off the first four satellites it acquires. There is no prioritization of one constellation over another. Simulations by Cao (2008) show that multi-frequency antennas have lower TTFF than single-frequency antennas. The best results in both mean TTFF and positioning performance are with multi-frequency, multi-GNSS receivers. In testing PPP-RTK convergence times with GPS-only and GPS+Galileo receivers, Yan & Zhang (2022) found that multi-GNSS receivers required less time for both horizontal and vertical components to converge. Although the inability to employ RTK methods for submarines at sea is already discussed, this study continues to show the benefits of multi-GNSS receivers. However, like with positioning accuracy, the benefit of a lower fixing time with multi-GNSS receivers also comes with the consequence of higher power demand (Garmin, n.d.a).

2.13.2 Multi-GNSS Options

The GLONASS constellation offers similar positioning accuracies to that of GPS (Larson & Wertz, 2005). GLONASS users have experienced positional accuracies between 4.5 and 7 meters, meaning the constellation's differences in satellite orbital inclination, satellite altitude, and software generate superior results to GPS only some of the time. The same can be said for BeiDou, which offers worldwide positional accuracies of less than 5 meters, although performance improves and even surpasses GPS in the Asia-Pacific region where more satellites are visible (Liu *et al*., 2022). Although GLONASS and BeiDou have similar performance results as GPS, and the system differences can offer certain advantages to users, the political uncertainties between Canada/Russia and Canada/China jeopardize the systems' reliability particularly for military operations. Although GNSS are one-way systems, in that the GNSS do not receive any information about the user's movements, users call for a certain level of trust and reliability on the GNSS owner for continuous and accurate positioning services.

Galileo, alternatively, was created by the European Union (EU) with which Canada maintains a strategic relationship (Verdun, 2021). The Canadian financial contributions to Galileo's development (Government of Canada, 2003) as well as a storied collaboration between Canada and the EU in aspects of foreign affairs, trade, security, environment, and business establishes the foundation of not only Galileo's reliability to Canadian users but also for system security. Altogether, Galileo is the most likely GNSS available today to be paired with GPS with the goal of increasing current PNT performance (Government of Canada, 2023a).

2.14 Galileo

Like GPS, Galileo offers continuous, all-weather positioning and timing services to its users through a network of 24 satellites in MEO. Unlike GPS though, Galileo was designed from the start in 2003 for civilian use versus being military-oriented

(Bowler & Wall, 2014). While Galileo would be an independent GNSS, it was designed from the start to also work in conjunction with GPS, offering augmented positioning services compared to those of just GPS (Grewal, 2020). Galileo was declared fully operational in 2020; as a newer system with superior positioning performance for all users, navigation system experts regard Galileo as superior to GLONASS, BeiDou, and even GPS (Hadas, Kazmierski & Sosnica, 2019; Van Do, 2021). Furthermore, Galileo has been proven to improve positioning performance when used in conjunction with other GNSS, adding value in LBS (EUSPA, 2014).

2.14.1 Galileo Background

Galileo is a GNSS owned by the European Union (EU) and operated by the European Union Agency for the Space Programme (EUSPA). Justification for the cost of designing, launching, and maintaining Galileo was to ensure that European governments and militaries could operate independently of other nations' GNSS which can be disabled or corrupted without warning (Bowler & Wall, 2014). In addition to three-dimensional low- and high-accuracy positioning for the public and authorized users respectively, Galileo was also designed to augment worldwide SAR capabilities.

In 2003, Canada contributed $11 million towards the development and validation of Galileo capabilities that lasted for the next three years (Government of Canada, 2003). Despite not being an EU-member, the partnership would allow Canadian companies to bid on elements of Galileo, which when fully functional, would benefit many aspects of Canadian life. In 2004, the EU formally agreed with the U.S. that Galileo would not only co-exist with GPS, but that Galileo would be compatible for any future collaboration or combined usage.

Following the first two test satellites launched in 2005 and 2008, and the first satellite destined to be part of the eventual constellation launched in 2011, European users began seeing Galileo signals in their navigation systems and mobile devices in October 2016. In December of that year, Galileo offered Early Operational Capability (EOC) which enabled GNSS research into potential benefits of the new constellation (Luo, Chen & Richter, 2017); however, Galileo's services could not be accessed by all users around the world, particularly in the U.S. (Barbeau, 2018). The American FCC prohibited receivers in the U.S. from accessing non-U.S. satellite signals. The fact that Galileo did not interfere with GPS and the prospect of delivering increased PNT solutions to its customers resulted in manufacturers supporting the EU and the introduction of Galileo in the US. The EU submitted for a waiver from this rule in 2013, with US manufacturers including Broadcom, Qualcomm, and T-Mobile voicing their support for Galileo to the FCC. The relatively easy integration of Galileo into existing GPS receivers would enable improvement in PNT solutions for their customers rather inexpensively and without concern for system security or solution degradation. It was only in November 2018 that the FCC approved a waiver to allow for certain Galileo signals (E1 and E5) to be used for non-federal PNT within the U.S., which included GPS augmentation (Trautvetter, 2018), which opened the door to multi-GNSS receivers that incorporated Galileo. That same year, Galileo consisted of 18 operational satellites that provided near-global coverage.

As of 2022, Galileo was relied upon by three billion people around the world for positioning services (Cozzens, 2022a). At the time of this writing, the International Civil Aviation Organization (ICAO) is expanding its Standards and Recommended Practices to include Galileo while avionics are starting to include Galileo as a standard positioning service (FAA, 2022).

2.14.2 Galileo Satellites

The Galileo satellites were developed and launched by the European Space Agency (ESA). Each Galileo satellite broadcasts navigation data in messages to be received by receivers. Galileo satellites transmit on multiple frequencies: E1 (1575.42 MHz), E5 (1191.795 MHz), and E6 (1278.75 MHz). Like GPS, the orbits and onboard clocks of Galileo satellites are similarly monitored from the ground. The ground segment can also successively update satellites with software upgrades, like those to the navigation message to improve system performance (EUSPA, 2022b).

Again mimicking the functioning of GPS, Galileo signals arrive at receivers at similar very weak power levels. Galileo clock and satellite data is available on cellular networks and the cloud for abbreviated fixing times. However unlike GPS, the European Commission ordered that Galileo be designed from the beginning to have quicker TTFF than GPS without the aid of assist concepts.

Galileo’s data messages, which house the clock and ephemeris data, are contained in four 2-second messages that repeat every 30 seconds (Hubert, 2022). However, only under ideal conditions is all pertinent data received in a single 20-second cycle (Hubert, 2022). In the event all four messages are not received, the 30-second cycle also includes a lower-precision descriptor message, while parity bits also provide error detection (Hubert, 2022). These differences from GPS mean a Galileo receiver should have all information needed for a fix within 30 seconds, with lower-precision ephemeris data within eight seconds for a fix three meters less precise (Hubert, 2022). The Galileo ephemeris is then valid for approximately four hours, while the low-precision descriptor is valid for ten minutes (Hubert, 2022).

Unlike GPS which only uses one signal, Galileo’s navigation message differs between the various services including Open Service (F/NAV and I/NAV messages), High-Accuracy Service (I/NAV message), and Public-Regulated Service (G/NAV message) which allows for optimization based on the user’s needs (Gutierrez, 2023). Each Galileo message is divided into sub-frames which can be further divided into pages containing the satellite's ephemeris and clock information (Pena *et al*., 2009).

2.14.3 Open Service

The Open Service (OS) offers dual-frequency positioning with one meter accuracy for all public users free of charge (Hadas, Kazmierski & Sosnica, 2019; Van Do, 2021). With similar design principles as GPS L1 C/A code, the OS was designed to work in conjunction with GPS to offer improved positioning performance compared to working with just GPS (Grewal, 2020).

The F/NAV signal is transmitted at a rate of 25 bps on the E5a-I frequency (Walter, Gunning & Blanch, 2016). The F/NAV message is 600 seconds long, comprised on twelve 50-second sub-frames that are made up of five 10-second pages (Walter, Gunning & Blanch, 2016). Each 50-second sub-frame contains the ephemeris, almanac, and clock data for positioning.

2.14.4 High-Accuracy Service

Originally, Galileo offered a subscription-based Commercial Service (CS) for higher performance than OS through additional encrypted signals (Grewal, 2020). Both OS and CS could be sent offline in exceptional circumstances, such as in a crisis, and are vulnerable to malicious interference such as jamming.

In 2023, Galileo's high-accuracy service (HAS) became operational replacing the CS. HAS enables worldwide PPP, offering horizontal positioning accuracy down to 20 centimeters for properly-fitted receivers (Cozzens, 2023b). The high integrity signal also reduces TTFF and risk to spoofing, increasing its robustness (Cozzens, 2023b). HAS is openly accessible to users around the world, who require a receiver for the HAS correction broadcast through the E6-B signal as well as internet connectivity (Cozzens, 2023b). These corrections reduce the positioning errors inherent with the Galileo OS and GPS SPS.

The I/NAV signal, which is also associated with the OS, is repeatedly transmitted at a rate of 125 bps on the E1-B and E5b-I frequencies (Walter, Gunning & Blanch, 2016). The I/NAV message is 720 seconds long, consisting of 24 30-second frames made up of 15 two-second pages. Like GPS, Galileo front-loads the ephemeris and clock data in the 30-second frame with the entire almanac spread over multiple frames, as seen in Figure 15. Unlike GPS though, Galileo transmits this data at five times the rate meaning a receiver receives all the data needed for a position fix in shorter period of time (Samson, 2011). While GPS's lower data rate is more advantageous in GNSS-challenged environments, it results in a slower TTFF and longer read time at the user segment. Not only is Galileo's data transmitted much quicker than GPS, but Galileo also repeats the clock data within each 30-second sub-frame. Each page also contains reserve bits for potential future upgrades (Walter, Gunning & Blanch, 2016).

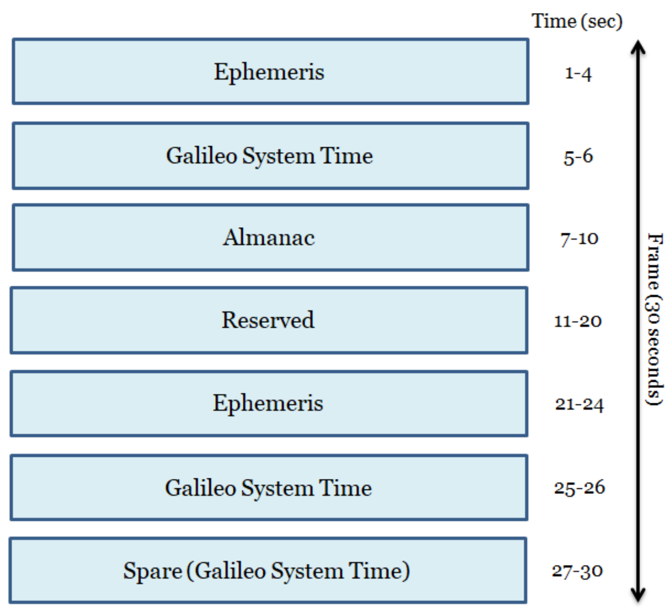

Figure 15. The Galileo I/NAV Message structure.

In 2023 the E1 OS I/NAV message was updated with three features intended to abbreviate device initialization and provide more robust service (Saines, 2023). The Reed Soloman Outer Forward Error Correction (RS FEC2) reduces the time for a receiver to retrieve Clock and Ephemeris Data (CED) by providing redundant data across the 30-second message. The unique RS FEC2 also augments the robustness of demodulation by enabling the automatic restoration of any corrupted data (Saines, 2023). These changes were intended to improve performance for both unassisted and assisted Galileo services (Gutierrez, 2023).

The reduced CED retrieval time enable faster data retrieval which results in lower TTFF, albeit with lower positioning accuracy as the receiver works with only a single I/NAV word (Cozzens, 2022a; Cozzens, 2022b; Gutierrez, 2023). The user is subjected to lower positioning accuracy until the receiver decodes the remaining I/NAV words with the full precision CED (Saines, 2023). These new signal architectures and capabilities deliver a quick course position estimate and also reduce the time required for a full accuracy solution, reducing TTFF for all OS users (Gutierrez, 2023; Saines, 2023). These I/NAV improvements are also expected to improve Galileo OS performance in challenging environments where satellite LOS is limited (EUSPA, 2022a).

2.14.5 Public Regulated Service

Galileo also offers encrypted positioning services to authorized users through the Public Regulated Service (PRS), which offers anti-jamming, anti-spoofing, and error-detection capabilities capability (Grewal, 2020; Luccio, 2023). Like GPS's PPS, the PRS is a controlled, secure, and encrypted (access-controlled) service only available to government-authorized entities. PRS is designed to offer continued and reliable PNT services in times of crisis, when the OS and CS can be turned off, degraded, or compromised.

The PRS signals are continuously broadcast on two frequencies – the 1575.42 MHz E1 and 1278.75 MHz E6. The use of wideband signals for PRS ensures the continued service despite any targeted or unintentional interference. Like PPS receivers with GPS, access is controlled through the encryption of signals and the member states maintaining control of decryption keys. Unlike GPS though which manages PPS decryption keys through the DoD, the responsibility of PRS key distribution falls to individual EU member state governments (Space News, 2008). Authorized PRS users are focused in security, including law enforcement, intelligence, emergency response services, customs, and coast guard, but also include sensitive strategic infrastructure such as telecommunications and energy. Civil organizations can also apply for usage through the EU's security policy and pay for their own PRS-enabled receiver (Space News, 2008). In 2008, a survey showed that 70% of prospective PRS users in Europe were already using GPS PPS receivers; their aspirations were to complement that existing capability with high-fidelity PNT with anti-jamming capability (Space News, 2008).

The process of acquiring and implementing Galileo PRS receivers is relatively straightforward for EU member states. A proposal shall be forwarded to the European Parliament who on a council's recommendation may approve individual nations' desire to use the encrypted signals. Each member state is then required to establish a "Competent PRS Authority" who takes responsibility for controlling and managing PRS receivers. While not just working for the distribution of PRS receivers, each member state is responsible for conforming to and upholding the stringent security standards associated with the unique receivers.

At the time of this writing, the use of Galileo for military users is in its infancy. The GEODE (Galileo for EU Defense) project is intended to bring PRS capability to military users by the end of 2026 (Cozzens, 2021). French-based Orolia was selected in 2021 to develop a standardized Galileo PRS-enabled receiver for EU member states' military applications (Bureau, 2021). These receivers would then be used for precise PNT of both crewed and uncrewed military vehicles, including UAVs, ships, and missiles, as well as ground infrastructure to support networks and cybersecurity (Bureau, 2021). The GEODE program also includes development of combined GPS and PRS compatible radiation pattern antennas (CRPA) as well as the design of PRS receivers for spacecraft (Cozzens, 2021).

2.14.6 Search and Rescue

In addition to PNT services, Galileo originally offered a Safety of Life (SoL) Service for users in distress. The I/NAV message contained a dedicated slot for return data from the rescuers which could alert the user that the call has been received (Anghileri, 2013). The SoL Service was available on the E1 OS signal, ensuring its widest availability to users (Pena *et al*., 2009). The service was discontinued in 2013, leaving available bandwidth in the data message structure for subsequent system and signal updates.

2.14.7 Galileo Performance Today

At the time of this writing, Galileo consists of 28 satellites with a plan to total 30 satellites (including 3 spares) (ESA, 2023) (Trimble, 2023). The EUSPA has publically committed to maintaining Galileo and improving its free and subscription-based PNT services (EUSPA, 2022a). While plans for future satellites are available online, their exact launch dates and capabilities are not guaranteed.

Like GPS, Galileo positioning performance continues to improve with changes in the data message structure, receiver technologies, and augmentation services. Galileo TTFF likewise benefits from assist data servers which also help reduce a receiver's power demand. A Secondary Synchronization Pattern (SSP) within the 2023 I/NAV update expedites resolution of clock uncertainty from assist channels, reducing a device's TTFF when operating with assist data (Saines, 2023).

2.15 GPS and Galileo Working Together

From the beginning of Galileo's design, the U.S. and EU were in talks to ensure the two independent systems would be compatible, interoperable, and interchangeable. The previously discussed benefits of multi-GNSS receivers (positioning accuracy, system integrity, and fixing time) are a result of the design decisions with regards to Galileo's satellite configuration, signal frequencies, data rates, power levels, and data message. While the similarities of the systems enable interoperability, it is the increased number of available satellites rather than the subtle differences in message format that cause improved convergence rates.

The improvement in positioning accuracy by integrating Galileo is most noticeable and beneficial in urban regions, where signals are highly attenuated and single-GNSS operations can become unreliable (Lawrence *et al*., 2017). Multi-GNSS operations on the other hand see an increased number of visible satellites which reduces the probability of signal loss (Bowler & Wall, 2014). Furthermore, the E5/L5’s unique signal shape helps receivers distinguish between real signals and those reflected off nearby buildings, reducing error from multipath effect.

When integrating GPS and Galileo in field tests, Luo, Chen & Richter (2017) experienced faster ambiguity resolution as well as better positioning accuracy. While both improved, testing also shows that integrating Galileo provided more accurate and more consistent performance in vertical positioning (altitude) than horizontal positioning (Luo, Chen & Richter, 2017). In GNSS simulations with short baselines and low ionospheric errors, single-frequency GPS+Galileo RTK receivers also out-perform dual-frequency GPS-only RTK receivers (Cao, 2008). Furthermore, dual-frequency GPS+Galileo out-performs triple-frequency GPS-only in terms of ambiguity resolution (Cao, 2008). Interestingly, the use of GNSS signals with the same frequency (GPS L1 and Galileo E1) provide greater positioning performance than two signals on different frequencies (Cao, 2008). Lastly, the integration of Galileo with GPS improved convergence times in PPP with Ambiguity Resolution – both in static and kinetic tests (Du, 2022).

Despite the similar orbital configuration of the two systems, the greater number of operational GPS satellites than Galileo satellites results in one to three more satellites available to users, as seen in Figure 16 (Hadas, Kazmierski & Sosnica, 2019). Having more GPS than Galileo in view was observed by Pandele *et al*. (2020) in their shipboard testing of GNSS receivers. As a result of having less available satellites, Galileo HDOP is worse than that of GPS, as seen in Figure 17. Slightly higher DOP values for Galileo were also observed by Pandele *et al*. (2020), which is directly attributed to the number of satellites available.

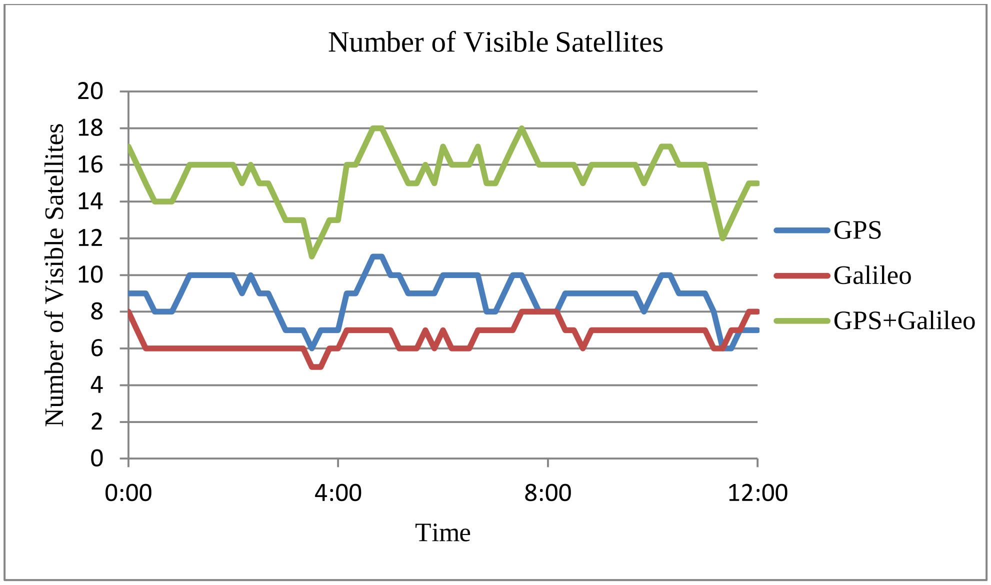

Figure 16. Number of visible GPS and Galileo satellites throughout a 12-hour period on 1 December 2023 off the coast of Halifax, Nova Scotia (42° 14' 40" N, 62° 55' 0" W).

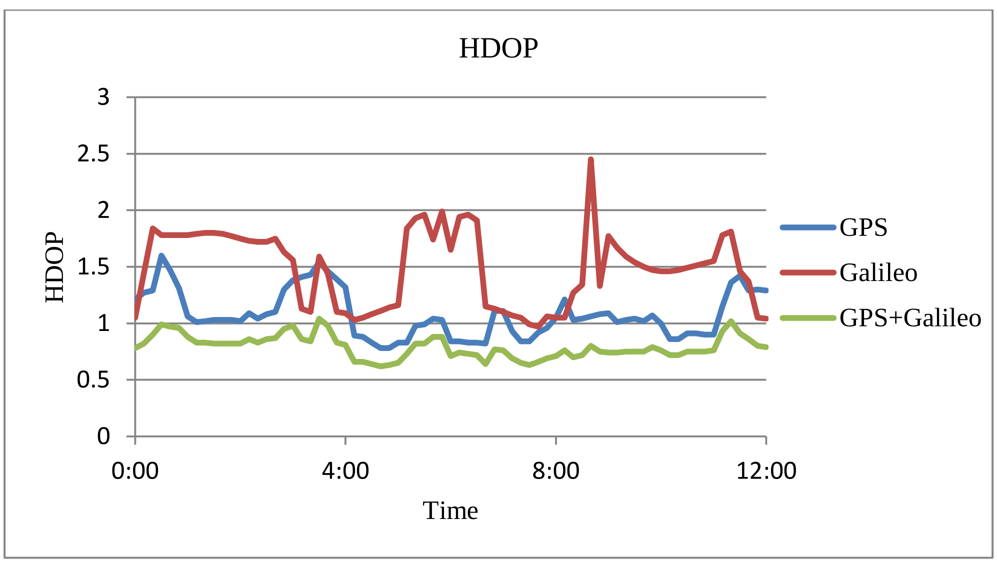

Figure 17. HDOP throughout a 12-hour period on 1 December 2023 off the coast of Halifax, Nova Scotia (42° 14' 40" N, 62° 55' 0" W).

However, owning an integrated GPS and Galileo receiver means that users see their satellite availability almost double, resulting in decreased and more stable HDOP values relative to users operating only with GPS. In order for GPS to increase the number of available satellites by one satellite to a user anywhere on Earth, the U.S. would need to launch four more satellites (Melgard, 2013); instead a more economical strategy is the pairing of GPS with other GNSS (signals of opportunity) to enhance satellite availability and reduce positioning errors, particularly atmospheric error from low-elevation satellites and multipath error in urban centers (Cao, 2008; Melgard, 2013). Compared to a GPS-only receiver, GPS+Galileo receivers offer a 10% improvement in positioning accuracy, with the benefits of integrating the two GNSS even more evident in urban canyons (Hadas, Kazmierski & Sosnica, 2019). Thus, GNSS interchangeability is especially important if a user is limited by infrastructure and/or has a high elevation cut-off angle (Melgard, 2013).

In terms of signal frequency, there is some overlap between GPS and Galileo (Table 5 and Figure 18) as both systems were specifically designed for interoperability (Luo, Chen & Richter, 2017; Pena *et al*., 2009). Full interchangeability[23] between the two systems is enabled by the use of Code Division Multiple Access (CDMA) as well as the shared frequencies of the L1 and E1 signals, and the L5 and E5 signals (Melgard, 2013). From the receiver's standpoint, use of the same frequencies enables use of the same radio frequency circuitry by both GPS and Galileo signals, saving space and material. However, since both signals are distinctly different, they require dissimilar signal processing in terms of decoding and demodulating processes.

Table 5. Comparison of frequencies between GPS and Galileo (Cao, 2008).

GNSS	Designation	Center Frequency
GPS	L1	1575.42 MHz
	L2	1227.60 MHz
	L5	1176.75 MHz
Galileo	E1	1575.42 MHz
	E5a	1176.75 MHz
	E5b	1207.14 MHz
	E6	1278.75 MHz

[23] Where navigation data from different GNSS are used to output a single positioning solution (Melgard, 2013).

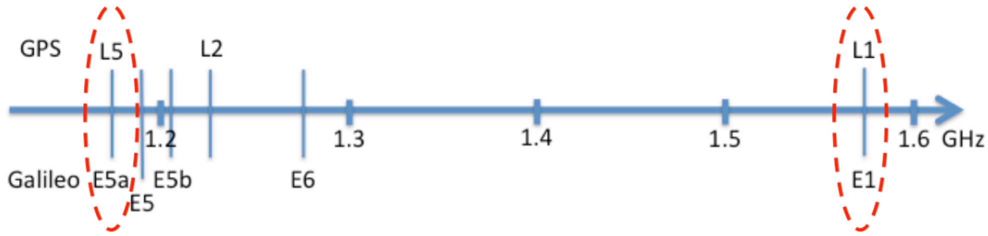

Figure 18. The use of some of the same frequencies facilitates the manufacturing of GPS+Galileo receivers (Luo, Chen & Richter, 2017; Melgard, 2013).

Both the GPS and Galileo navigation data messages contain the ephemeris and clock information needed by a receiver to solve the navigation equations to determine location (Pena *et al*., 2009). The difference in where that information lies in the messages and their repeatability results in GPS and Galileo performing differently. There are four major differences between the GPS and Galileo signals, which may cause a GPS and a Galileo receiver to perform differently.

First, the GPS and Galileo signals are made up of data channels and pilot (non-data; instead used for tracking) channels. While the data channels carry the ephemeris and clock data, the pilot channels contain the ranging and timing information in the PRN codes. Although both the GPS L1C and Galileo E1 OS signals are received by antennas at the same power level, the different power distributions between the data and pilot channels of the two constellations (outlined in Table 6) result in the Galileo data arriving at twice the power[24] of the GPS C/A code (Pena *et al*., 2009). Galileo's stronger signal enables a shorter $T_{acquire}$ but brings the risk of false acquisitions. Second, the Galileo symbol transmission rate (250 symbols per second) is 2.5 times that of GPS (100 symbols per second) (Table 7) (Pena *et al*., 2009). Third, the structure of the data messages, including the size, frequency and layout of information is different. While the GPS minimum read time is 18 seconds, the minimum read time for Galileo is 14 seconds (Samson, 2011).

[24] Higher power results in a higher C/N_0, or higher amplitude of symbols with regards to noise.

Table 6. Relative power distribution on the GPS and Galileo data messages (Pena *et al*., 2009).

	GPS	Galileo
Data Channel	25%	50%
Pilot Channel	75%	50%

Table 7. Comparison of the GPS and Galileo message parameters (Pena *et al*., 2009).

Parameter	GPS	Galileo
Frame Period	30 seconds	30 seconds
Data rate	C/A: 50 bit/sec L1C: 100 bit/sec L2C: 25 bit/sec L5: 50 bit/sec	E1: 250 bit/sec E5a: 50 bit/sec E5b: 250 bit/sec
Frame Period	C/A: 750 seconds L1C: 18 seconds L2C: 168 seconds L5" 84 seconds	E1: 720 seconds E5a: 600 seconds E5b: 720 seconds

Other than the ephemeris information, system time and clock correction are the other requirements for a receiver to estimate its position. In order to do that though, the signals of both systems must be synchronized with the receiver's internal clock. Both GPS (GPS Time) and Galileo (Galileo System Time) maintain their own independent system times which are defined by their respective ground segments. Both data messages contain correction information particular to that GNSS (Melgard, 2013). The difference between GPS Time and Galileo System Time is defined as the GPS-Galileo Time Offset (GGTO). Although Galileo System Time is usually within 50 nanoseconds of GPS Time, a receiver needs a common time reference in order to use signals from both constellation's satellites. Otherwise, a fifth satellite[25] can be used to estimate the GGTO. If the GGTO is not available and a fifth satellite is not an option, a pre-set GGTO value can be used

[25] Not always available in challenging environments.

albeit with the risk of it being out-dated which could lead to positioning errors of up to 15 meters (ESA, 2013; Melgard, 2013).

In order to achieve interoperability, Galileo satellites broadcast the GGTO as part of its data message (ESA, 2013). Since the GGTO is broadcast every 30 seconds, this method places more reliance on the entire navigation message since the receiver must wait 30 seconds for all the required data for both systems to work together. With the time offset known, a receiver can then align all the satellite measurements and reference them to the receiver's internal clock (Melgard, 2013). This way, only four satellites from either constellation are needed for a GPS+Galileo position fix.

The differences between the GPS and Galileo data messages leads to the following conclusions found in research. First, longer information frames equate to better signal performance. However, since an error in decoding individual words renders that whole message useless, shorter messages enable the receiver to restart decoding a new message from that satellite quicker (Pena *et al.*, 2009). Furthermore, the demodulation of ephemeris data for GPS signals is superior in terms of Bit Error Rate, Word Error Rate, and Ephemeris Error Rate (Pena *et al.*, 2009).

The thoughtful design of Galileo alongside the existing GPS makes the two systems both interoperable and interchangeable. A receiver operating with the same frequencies on different constellations (such as L1 and E1) provides better positioning performance than single-constellation dual-frequency, meaning that there is a greater advantage from increase satellite availability and geometry than frequency diversity (Cao, 2008). The newer L5 and E5a signals are wideband and thus provide greater mitigation against noise and multipath error. The use of not only multi-GNSS but also multi-band maximizes the receiver's potential in both positioning performance and system security. Altogether, users should capitalize on multi-GNSS capabilities if possible.

2.16 Combined GPS+Galileo Capability for Submarines

Due to the differences in the constellations and navigation data messages, a combined GPS+Galileo receiver could reduce a submarine's TTFF, TTFGF, or both. A more rapid fix that meets the submarine's accuracy threshold would enable the submarine to update its navigation systems quicker upon surfacing, allowing the submarine to submerge again more rapidly than if using a GPS-only receiver. Thus, since the RCN's Trimble receivers are manufactured in the United States, the FCC's 2018 approval to allow Galileo augmentation of GPS enables the incorporation of Galileo into military receivers, ultimately allowing the RCN and other users within the Canadian Armed Forces to operate with Galileo.

2.17 This Thesis

Submarines rely on their stealth for mission success, and thus remain fully submerged underwater as much as practicable. Upon returning to PD or surfacing, acquiring a good GPS position estimate is critical to the submarine's continued safe navigation. A low fixing time is highly favourable, limiting the submarine's exposure and enabling the submarine to dive again soonest if needed. Operating outside the range of terrestrial navigation resources specifically designed to reduce TTFF means that submarines must rely on satellites entirely for all navigation data.

To date, GNSS research has focused on the three primary benefits of multi-GNSS receivers: improved positioning accuracy, enhanced system reliability, and lower TTFF. However, there is limited research on convergence rates of navigation solutions. At the time of this writing there are no available studies on the timeliness of accurate positioning immediately following cold or warm starts on surfacing submarines. With the submarine concerned with expediting solution convergence vice TTFF, this research introduces the novel measurement of TTFGF.

TTFF and convergence rate are the two factors that will help reduce a submarine's TTFGF, and they can be reduced by exploiting signals of opportunity from existing GNSS (Anghileri, 2013; Yan & Zhang, 2022). To the submarine's benefit, using Galileo is an opportunity to reduce its TTFGF with no additional subscription fee and while exerting little change to existing submarine infrastructure. However, only limited studies have examined GNSS positioning in the maritime environment (Alkan & Ocalan, 2013), while none have examined convergence rates in the maritime environment.

This thesis will investigate whether a multi-GNSS receiver provides a faster convergence rate, reducing the time between a submarine's antenna exposure/receiver initialization and the first "good fix" compared to a GPS-only receiver. To do this, we need to analyze the timestamp of the position estimate's error following warm starts. If a multi-GNSS receiver could reduce the wait time for a first "good fix," this type of receiver could add value to submarine navigation.

Chapter 3

Methodology

Currently the submarines of the RCN operate exclusively with GPS, with no augmentation from other GNSS to either improve positioning accuracy or reduce fixing times. Without the possibility of terrestrial infrastructure to provide augmentation services and with the lack of worldwide coverage by SBAS, submarines are left with the option of achieving low fixing times through signals of opportunity from other GNSS. With Galileo fully operational as of 2022, the system could be used to reduce a submarine's TTFGF upon reaching the surface of the water after a dive. This chapter will outline experimentation to compare TTFGF values between a GPS-only receiver and a combined GPS+Galileo receiver, which could provide reduced fixing times for submarines.

3.1 Data Collection

GNSS research primarily occurs with either real world data or simulated data – the differences between which are summarized in Table 8. With simulated data, a signal simulator replicates the signals that would be generated by the satellites and received based on receiver positioning, speed, time, and date, along with other settings such as antenna altitude. More complex simulators allow for the input of obstacles and signal interference to replicate the real-world user's positon as much as possible. The limitations of the Skydel GNSS simulator with TTFF research were highlighted in the research by LeVeel (n.d.). The researcher used a GNSS simulator to compare warm start fixing times of GPS and Galileo receivers, showing quicker first fixes with Galileo. However, GNSS simulators along with programs such as "GNSSLogger" and "GPSTest" that enable basic GNSS research only state the time of the first fix and present no information on the quality (accuracy) of those fixes.

Real-world testing, on the other hand, generates realistic results, showing how particular receivers actually perform with all the factors that affect TTFF and GPS positioning. Instead of relying on the one receiver paired with a simulator, the collection of real-world data with a variety of receivers can help researchers differentiate how different receiver architectures and capabilities perform. Manufacturers of receivers and Kalman filters will often show potential customers real-world data to prove performance and specifications. Through 213 studies reviewed by Siemuri *et al*. (2022) for machine learning techniques in GNSS, the authors highlight that 187 studies used real GNSS data while only 13 studies were based on simulated studies.[26] Although the purchase of individual receivers can be costly, and the travel and data collection can be time-consuming, the collection of real-world data is beneficial in the comparison of receiver performance- which is the purpose of this thesis.

Table 8. Comparison of real-world and simulated GNSS data collection.

Aspect	Real-World Data	Simulated Data
Procedures	▪ Uncontrolled environment	▪ Controlled environment
Satellite Signals	▪ Satellite signals are variable, depending on satellite positioning and satellite availability ▪ Receiver subjected to the number of signals broadcast by GNSS satellites within its FOV	▪ Customizable, controlled environment ▪ Limited by the number of signals that can be generated
Repeatability	▪ Non-repeatable	▪ Repeatable
TTFF Data Precision	▪ seconds	▪ one hundredth of a second
Cost	▪ Low cost hardware ▪ Higher cost, particularly for any travel to remote locations and testing on high-velocity vehicles	▪ High cost for simulator setup ▪ Lower cost for testing

Due to the sensitivity of submarine operations however, a comparison of real submarine positioning data cannot be conducted for an academic thesis intended for open discussion. However, the submarine's circumstances of intermittent connectivity and remote maritime environment can be replicated in order to compare the performance of two receivers following a warm start.

3.2 Test Apparatuses

According to Table 4, the methods available to reduce a receiver's TTFF are Assist Data, SBAS, and multi-GNSS. In order to verify how well multi-GNSS can reduce a submarine's wait time for a first "good fix," the receivers being tested will have

[26] The remaining 12 studies used both real and simulated data.

GPS and multi-GNSS capability, while not having the capability of connecting either to assist data or SBAS.

Two civilian GNSS devices, one GPS-only and one multi-GNSS (GPS+Galileo) will be used to assess positioning accuracy with time following a warm start. Different devices from different manufacturers will have proprietary software that impact T_{boot}, $T_{acquire}$, $T_{bit\text{-}synchronization}$, and $T_{compute}$. Therefore selecting two devices not only from the same manufacturer but also the same device family will reduce differences in sensitivity and signal processing as much as reasonably possible for this comparison.

The apparatuses for this study are manufactured by Garmin Ltd, an American-based but Swiss-domiciled company which specializes in GPS solutions for vehicles and outdoor activities. The selected devices are the Garmin ZŪMO 396 and the Garmin ZŪMO XT. These specific devices were selected for their ease-of-operation, their varied sensors configurations (satellite constellations), and their similar readout of positioning accuracy that will be analyzed for this research. Both units present real-time positioning accuracy readout, representing the receiver's positioning performance. The estimated error of Garmin receivers is horizontal positional error (HPE), listing the CEP with 2-sigma (95^{th} percentile) accuracy value. Thus for a 10 meter accuracy readout, the receiver is within 10 meters of its estimated position 95 percent of the time.

3.2.1 Garmin ZŪMO 396

The Garmin ZŪMO 396 is a GPS-only navigation device specifically designed for motorcycles, although it can be used for any route planning and monitoring. Although designed for terrestrial navigation and not marine navigation, it still estimates its position based on the same satellite navigation principles as the submarine's receiver that is outlined in Chapter 1. A hidden menu[27] enables the user to manually initiate a warm start, like the submarine would experience when exposing the GNSS antenna above the water surface after a dive. The ZŪMO 396 features a GPS positioning information refresh rate of 1 Hz, aligning with per-second data collection for this research.

3.2.2 Garmin ZŪMO XT

The Garmin ZŪMO XT is the successor to the ZŪMO 396, and as such, is also a navigation device designed for motorcycles and other off-road vehicle users. The ZŪMO XT is compatible with both GPS and Galileo satellites, offering increased positioning accuracy. Operating on the E1 frequency, the ZŪMO XT receives the I/NAV message for high-accuracy positioning.

[27] The menu is not discussed in any owner's manual, but was found by users and is discussed online.

As part of the upgrade from the ZŪMO 396, the ZŪMO XT features a 10 Hz calculation rate for satellite data which enables more accurate positioning for high-dynamic vehicles. The XUMO XT can also be paired with in-Reach devices for location sharing via the Iridium constellation as an additional safety mechanism.

The ZŪMO XT was selected for this research since it is a multi-GNSS receiver, capable of receiving both GPS and Galileo signals at the same time. This capability will enable a performance comparison with the GPS-only ZŪMO 396.

3.2.3 Device Comparison

Both the ZŪMO 396 and ZŪMO XT are satellite-only receivers; the devices do not have the capability to receive assist data. While both devices feature Bluetooth and WIFI capabilities, these features are not linked to the reception of assist data. Instead Bluetooth enables hands-free calling (when paired to a mobile phone) and voice directions to a helmet/headset while WIFI allows the user to download maps and software updates without a dedicated computer.

In terms of the device's frequencies, Wu, Guo & Zheng (2020) found that the Galileo E1 provides equal or even superior positioning performance than GPS L1. Although single-frequency receivers exhibit greater positioning errors from ionospheric interference and noise, both receivers will be impacted equally throughout testing, which will reveal any benefits from integrating Galileo with GPS. Table 9 compares other foundational specifications for both receivers, made available online by retailers. More in-depth engineering specifications, including chipset details, number of channels, rated TTFF, and sensitivity that could be used in this analysis of TTFGF are not openly available.

Table 9. Comparison of the Garmin ZŪMO 396 and ZŪMO XT (GPS Central, 2023).

Parameter	Garmin ZŪMO 396	Garmin ZŪMO XT
Release Date	11 April 2018	4 March 2020
Sensor(s)	GPS	GPS & Galileo
Signal Frequency	L1	L1 & E1
Update Rate	1 Hz	10 Hz

Both the ZŪMO 396 and ZŪMO XT output positioning accuracy (Horizontal Positioning Error) on a similar screen as seen in Figure 19, while also displaying the satellites currently in-view, their relative geometry, and their respective signal strengths. Not advertised either online or in the owner's manuals are how many channels each device has, which dictates how many satellites it can track simultaneously. However, in the tests run the ZŪMO 396 was seen to connect to a maximum of 12 satellites while the ZŪMO XT connected to a maximum of 22 satellites. Although some GNSS receivers allow the user to manually connect and disconnect from particular GNSS, the ZŪMO devices do not have the option, meaning two ZŪMO XT could not be used for the GPS and GPS+Galileo tests.

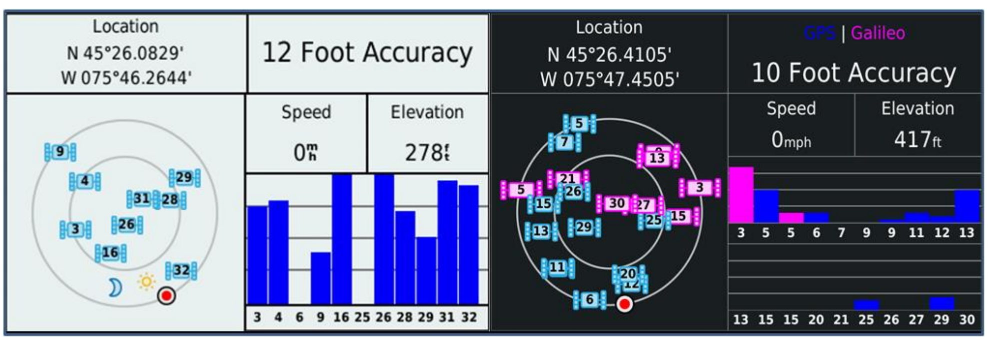

Figure 19. Screen captures of the Garmin ZŪMO 396 (left) and ZŪMO XT (right). GPS satellites are represented in blue and Galileo satellites are represented in pink.

3.3 Methodology

A series of 25 tests were conducted over multiple days and weeks to evaluate the positioning performances of a GPS-only and GPS+Galileo receiver following warm starts when the receiver is stationary. Spreading the tests over multiple weeks and also different times of day ensured varied satellite availability and geometry, as the submarine would expect over the course of a multi-week/month operation.

There are two principles in this data collection: (1) replicate the submarine's conditions as much as reasonably possible and (2) the comparison of solution convergence demands that both receivers operate under the same conditions for each test.

To ensure the satellite signals, receiver FOV, relative satellite positioning (no geometry difference), and satellite availability do not play a role in varying the comparison of positioning accuracy, the receivers will be co-located and be initiated at the same time. Prior to all data collection, the memories of both devices were cleared and the units returned to their factory/default settings. The settings of both devices, as well as the data collection process, will be identical throughout the data collection. It is therefore expected that both receivers will perform as per the manufacturer's specifications.

Since this thesis is focused on submarine navigation, an important aspect of testing was to replicate the submarine's open-ocean environment as much as possible. Therefore all tests were conducted in wide open areas where the receiver's had the best FOV possible, reducing the risk of multipath errors and LOS obstructions as much as reasonably possible. The Garmin receivers were also placed at an altitude of 1-meter above the ground, again replicating the submarine's antenna conditions when at PD.

Both receivers have a hidden menu that enables the user to manually force a warm start, where the receiver acquires the visible satellites based on the stored almanac data. Following manually-initiated warm starts, the positioning accuracy of both receivers was recorded at a rate of 1 Hz for a period of three minutes. Three minutes was seen to provide sufficient time for both devices to achieve not only a first fix but also good fixes before reaching steady state. Since the devices will already be powered-on and operating when the warm starts are initiated, T_{boot} can be removed from each device's TTFF. Thus the boot time and software loading of the individual devices is not a factor in this comparison of GPS TTFF and GPS+Galileo TTFF. With the devices already booted for this data collection, Equations (4) and (5) can be reduced to:

$$TTFF = T_{acquire} + T_{bit-synch} + T_{nav\ data} + T_{compute} \tag{6}$$

$$TTFGF = T_{acquire} + T_{bit-synch} + T_{nav\ data} + T_{compute} + T_{converge} \tag{7}$$

3.4 Assumptions

In order to keep this thesis and its results unclassified, the receivers tested will be non-encrypted, receiving the civilian signals of both GPS and Galileo. PPS receivers used by the military offer more accurate positioning and better signal security through the more precise P(Y) code. Although military receivers still need to first acquire and lock onto the C/A code before acquiring the P(Y) code (resulting in longer TTFF), military receivers can acquire satellite signals faster than civilian receivers due to their higher jammer-to-signal ratio (which can be ten times higher) (Schmidt, 2015). In their research on TTFF, Lachapelle and Rao (2012) assumed comparable start times for both military and civilian GPS receivers under open sky conditions. Since civilian and military receivers should exhibit similar satellite acquisition times in non-denied GNSS environments like in open-ocean conditions, the same assumption will be made the this research.

In these tests, the dominant sources of error are ionospheric delay and measurement noise. With testing conducted in open spaces for full FOV, the risks of poor positioning performance due to multipath error and noise are as low as reasonably possible. Still, in order to compare the receiver accuracies against each other, it is assumed that throughout testing, any significant sources of error affected both receivers equally. This includes any common-mode errors (satellite biases, atmospheric, ephemeris, etc.) that would affect both receivers similarly as well as non-common-mode errors (receiver bias, noise, multipath, etc.) which would affect the two receivers differently (Rahman, 2022).

It is also assumed that there was no local GNSS interference at the time of testing, which would affect the receivers' performance.

3.5 Test Conditions

All receiver tests were performed in the vicinity of Ottawa, Canada (45.4° North Latitude and 75.7° West Longitude) in December 2023 and January 2024. Ottawa is of similar latitude as Esquimalt, British Columbia (48.4° North) and Halifax, Nova Scotia (44.6° North) as seen in Figure 20 - the two naval bases that support RCN submarine operations (Government of Canada, 2024). With DOP and satellite availability influenced by latitude due to the Earth's rotation relative to satellite motion, Ottawa provides similar GNSS satellite skyplot conditions that submarines would experience in the waters near Halifax and Esquimalt, as well as exercises and patrols in the North Atlantic and Pacific Oceans (Hunt, 2020).

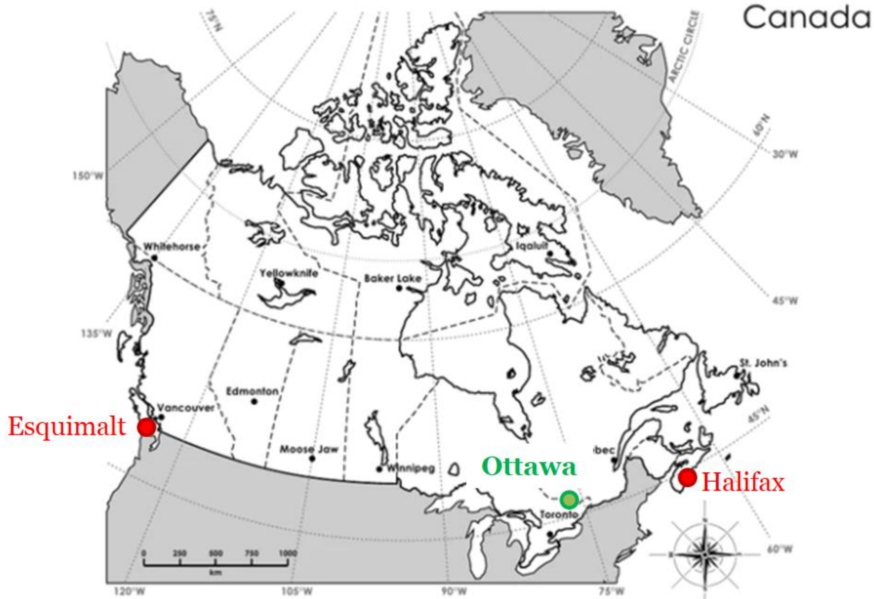

Figure 20. Test location of Ottawa at similar latitude as Esquimalt and Halifax.

3.6 Test Limitations

In using real-world data to test this hypothesis, there are aspects of satellite navigation and TTFF that cannot be controlled. The results presented in this study are subject to the satellite availability, navigation data messages, satellite geometry, and all sources of error that were present at the time of testing. Changes to the number of satellites, their orbital configuration, and their navigation data message structures would alter the results and recommendations presented hereafter. The results are also bounded by the architectures of the selected receivers. Thus the exact results cannot be attributed to GPS and GPS+Galileo receivers of other manufacturers.

The availability of GNSS receivers on the commercial market also limits how closely this study can simulate submarine operations. While the submarine relies on dual-frequency military receivers for increased accuracy, faster P(Y) acquisition, and enhanced error reduction, only single-frequency receivers were available by Garmin at the time of testing. Discussions online speculate the inclusion of the L5 frequency for reduced position error primarily for wearable athletic-tracking devices that are subject to high dynamics. The concern is the use of multi-band on smaller devices will both increase cost and power consumption.

Lastly, it is not expected that testing on land would have adverse effects on the results compared to testing on the ocean surface. For simplicity and cost-effectiveness, these tests were performed on land in open area conditions.

Chapter 4

Results

This thesis investigates whether the use of Galileo signals integrated with GPS could reduce the wait time required by a submarine to reach the GPS/INS positioning accuracy threshold. As discussed in Chapter 2, one possible method of reducing TTFF is taking advantage of existing signals of opportunity; these signals of opportunity from space could similarly be used to expedite the convergence rate and reduce the TTFGF. Since Canada operates predominantly with American combat equipment and has strong diplomatic and strategic relations with the EU, an RCN submarine could use Galileo in conjunction with GPS since the two systems are interoperable. Galileo has demonstrated higher positioning accuracy than GPS in certain conditions, and along with its continuous, all-weather, and compatible signals, it could be used to augment submarine PNT following a warm start when surfacing.

4.1 Solution Convergence Results

Since RCN submarines currently operate with GPS receivers, the first data set (GPS-only) is the base scenario and will be used to compare the performance of the multi-GNSS receiver. Serendipitously, the best conditions for a GNSS receiver are a full FOV with open-sky conditions, minimal noise, and limited electromagnetic interference – precisely the circumstances encountered by submarines in open waters. These circumstances enable the submarine receiver to lock onto whichever satellites are in the sky, maximizing the number of potential satellites for a quick and accurate position estimate.

Both the GPS-only receiver (Figure 21) and GPS+Galileo receiver (Figure 22) consistently produced a first position estimate within the first 10 seconds after their respective warm starts. As expected, the first fixes were always inaccurate compared to the receivers' long-term (2 minutes+) positioning accuracy. Following the first fix, the solutions converged over time as more satellites were acquired, more data was downloaded, and errors were corrected. The average positioning accuracy over the 25 tests per time interval is plotted in Figure 23 along with standard deviation bars.

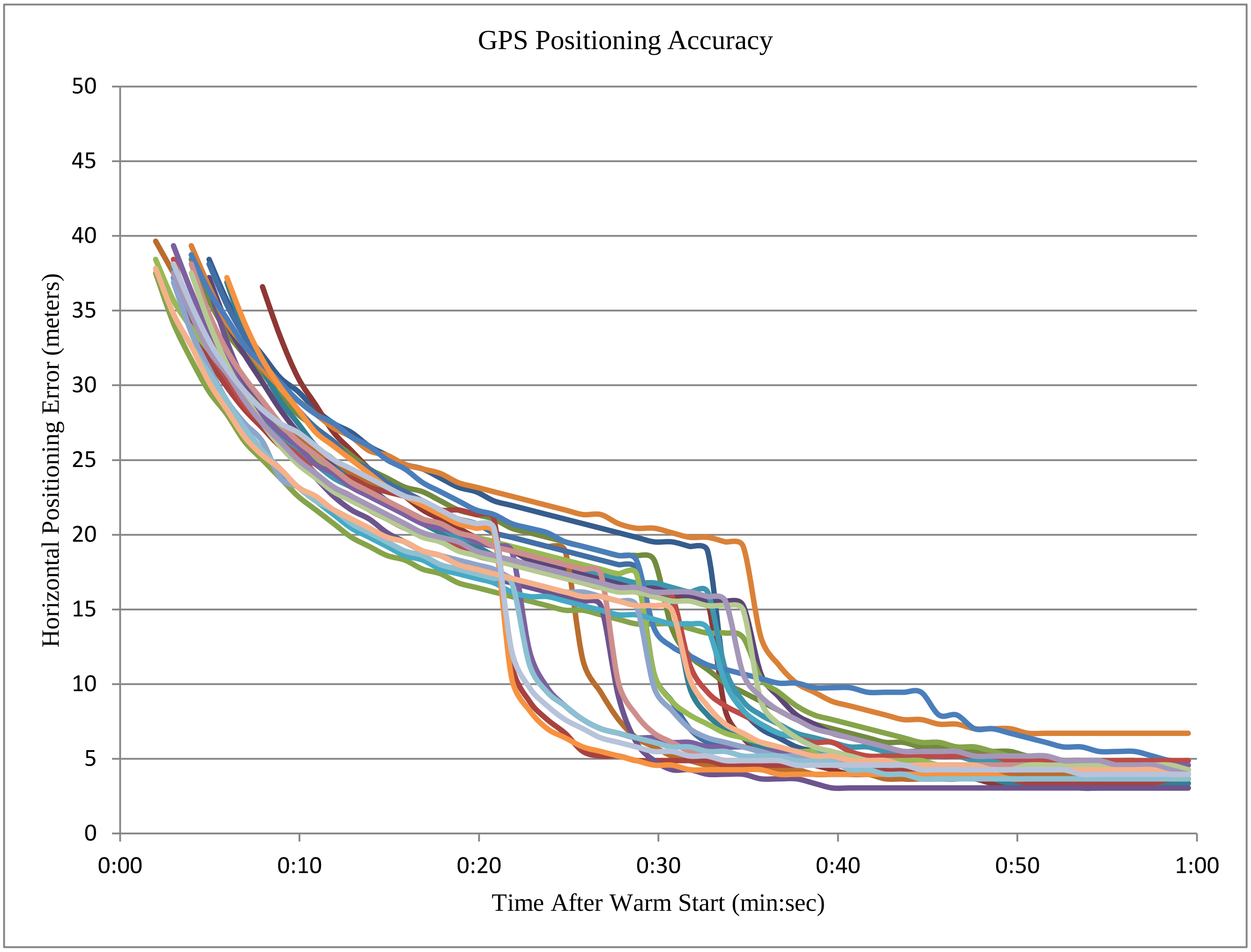

Figure 21. GPS solution convergence after 25 warm starts.

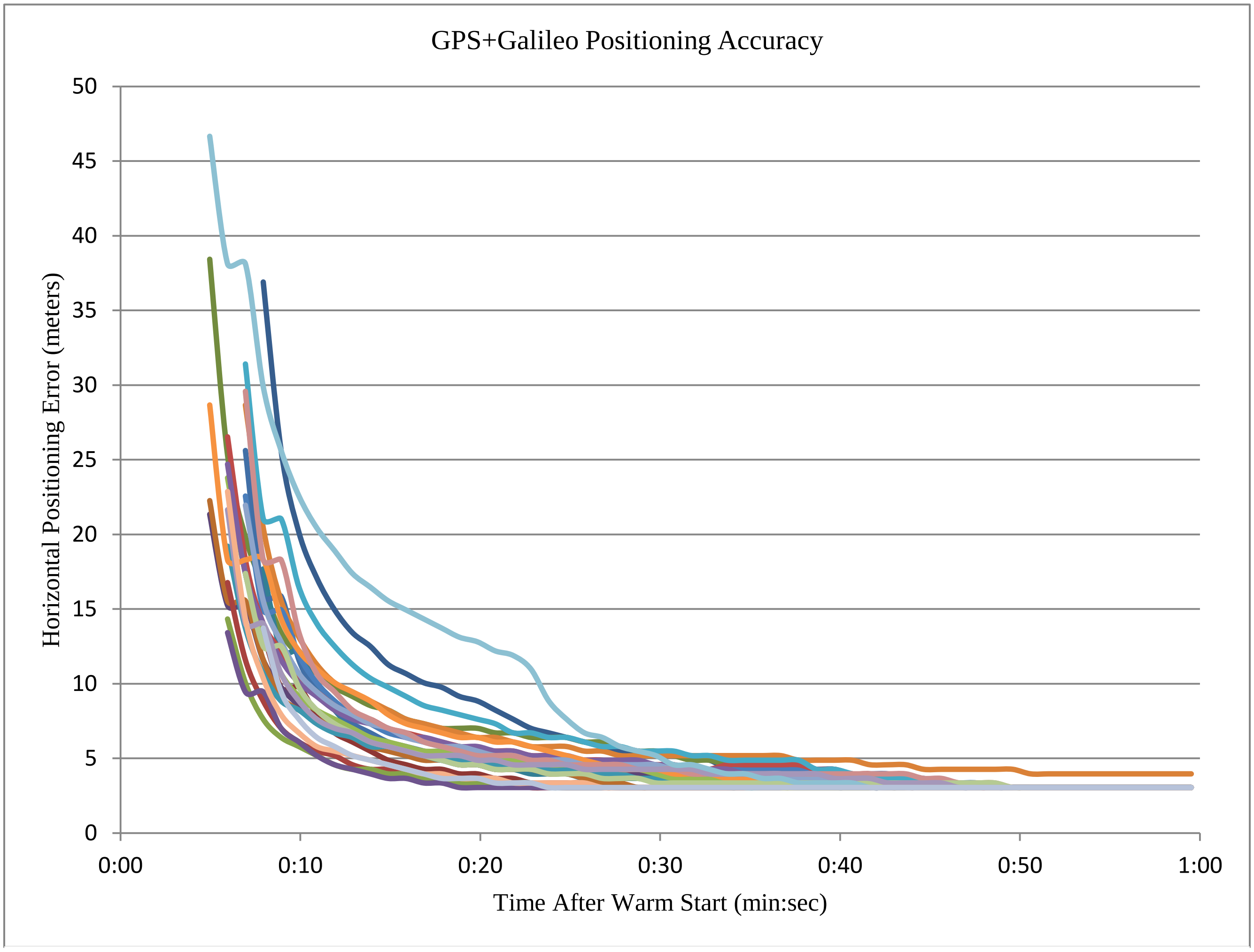

Figure 22. GPS+Galileo solution convergence after 25 warm starts.

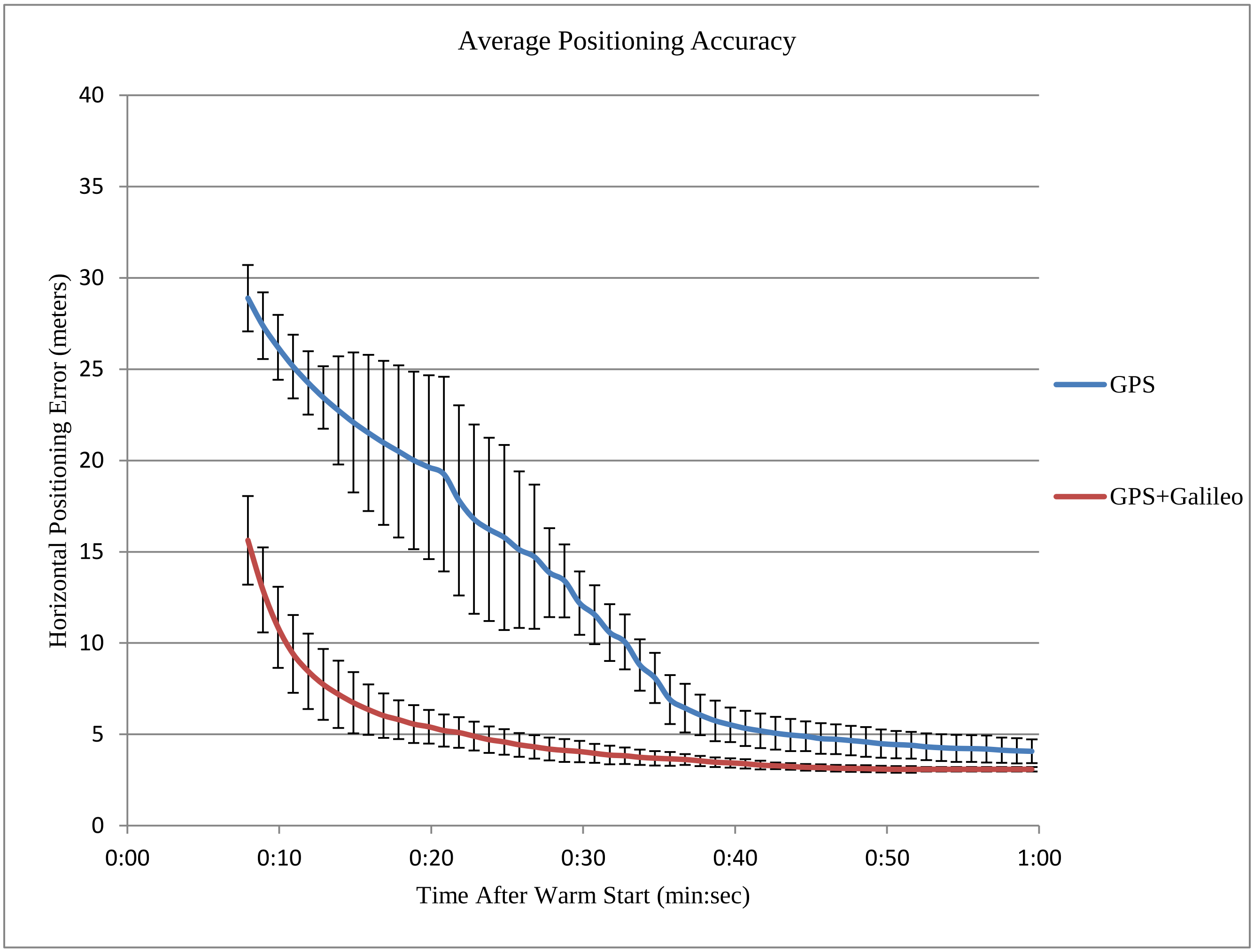

Figure 23. Comparison of average GPS and GPS+Galileo convergence with standard deviation bars.

4.2 Dynamic Testing

Studies show that GPS positioning accuracy decreases with increased receiver speed due to elevated noise levels and reduced data quality, which both affect satellite acquisition and data download (Marvel, 1998; Rahemi & Mosavi, 2021; Schmidt & Phillips, 2010). Aircraft also frequently experience carrier loss, which can lead to extended satellite re-acquisition times and degraded positioning performance. Since a submarine receiver's convergence rate may be impacted by the submarine's speed through the water, it is worth investigating whether receiver speed affects the convergence rate and TTFGF between the GPS-only and GPS+Galileo receivers. To verify if the submarine's speed plays a role in solution convergence and TTFGF, the same methodology for static testing was repeated for both receivers in 12 tests in various directions of travel at a speed of 22 kilometers per hour (12 nautical miles per hour (knots)), which is the maximum speed of the VICTORIA Class submarine when surfaced (Government of Canada, 2024).

In GNSS testing, dynamic tests are more complex to assess than static tests due to non-standardization (Lim *et al*., 2019). Satellite visibility, atmospheric errors, and noise all vary with the receiver's position, meaning that their influence on solution convergence and positioning performance would change over the course of dynamic testing. Furthermore, changes in the vehicle's (and thus the receiver's) trajectory and/or attitude can cause changes in GNSS signal quality.

To assess the impact of the submarine's motion on satellite acquisition and solution convergence, the same methodology was repeated on a vehicle travelling in a straight line at 22 kilometers per hour. Like the static tests, the test runs were conducted at different times of day and in different weather conditions. Since the submarine operates in open-ocean conditions, the dynamic test runs were performed in different directions of travel.

It was not expected that the movement of the receivers would have any significant impact on positioning for three reasons: first, the submarine's speed is much lower than that of cars and aircraft, which are typically used as platforms for dynamic GNSS testing. Second, the 2-minute window of this research's data collection is not long enough to experience noticeable changes in satellite and environmental conditions. Lastly, when surfaced or at PD, the submarine usually maintains a straight course at a steady speed when on the surface, negating the chance that changes in antenna direction of travel and attitude could impact its performance.

The results show the positioning performance of the GPS-only receiver (Figure 24) and GPS+Galileo receiver (Figure 25) after warm starts under dynamic conditions (20 km/h). The average positioning accuracy over the 12 dynamic tests per time interval is plotted in Figure 26 along with standard deviation bars.

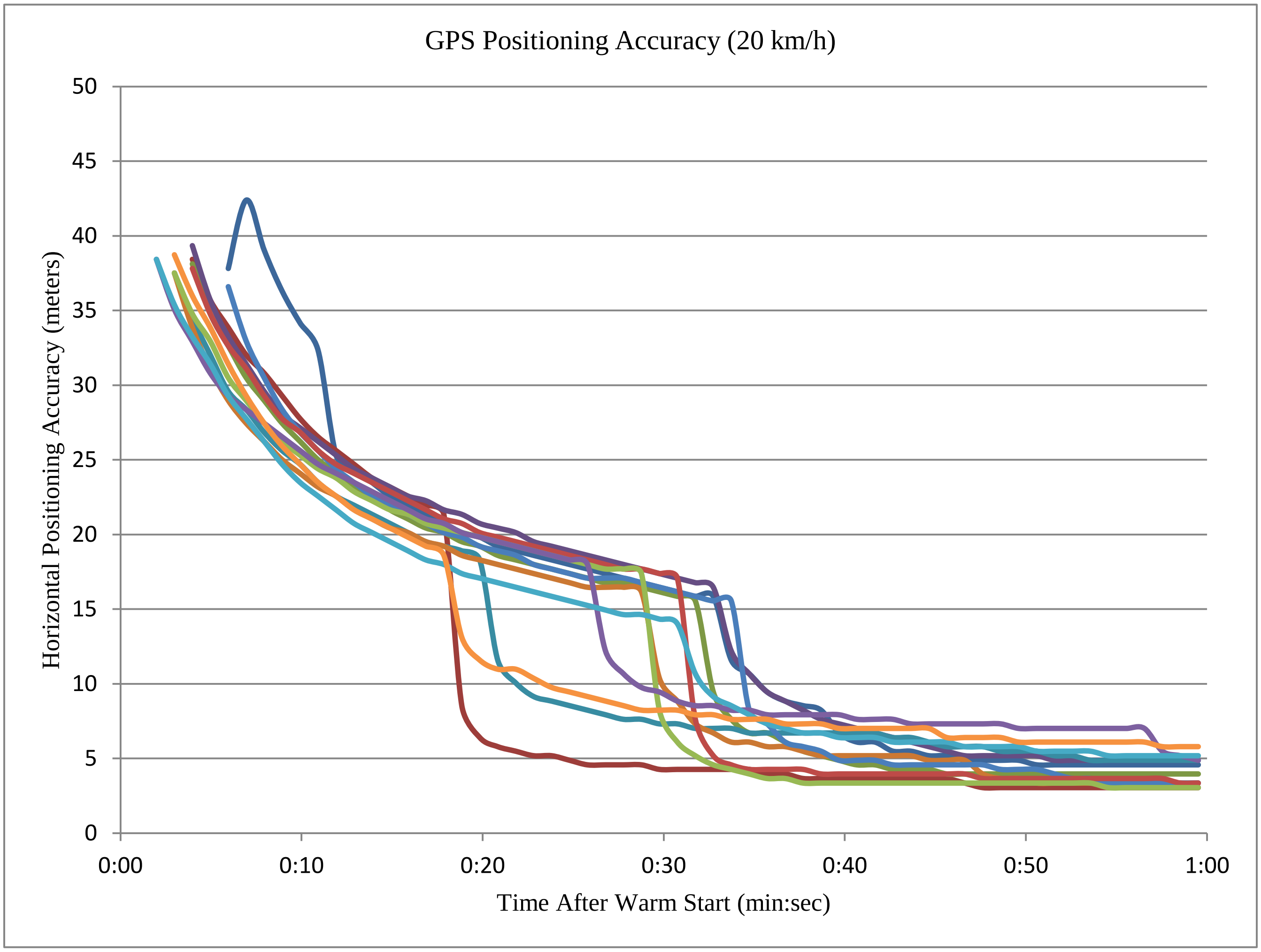

Figure 24. GPS solution convergence after 12 warm starts at 20 km/h.

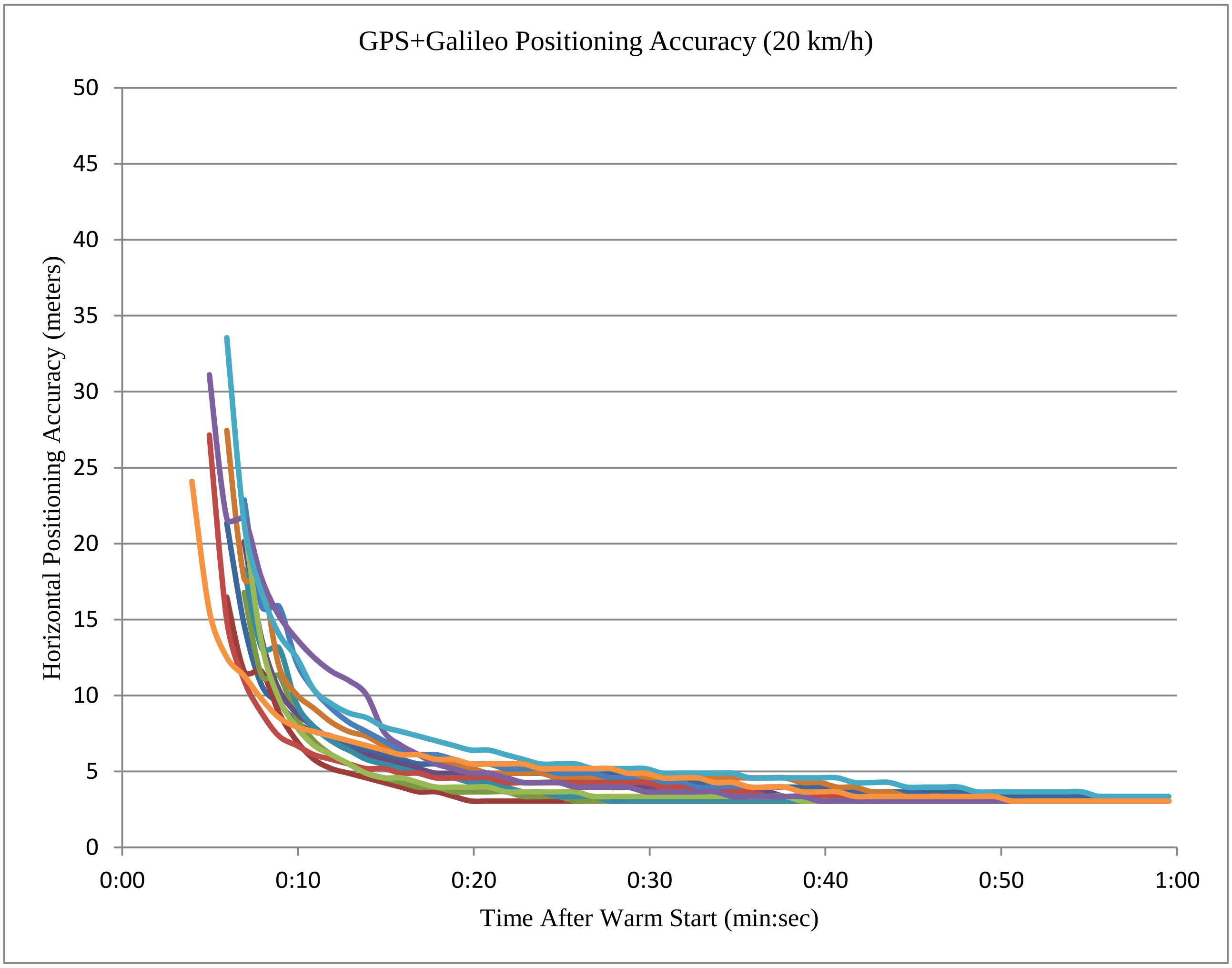

Figure 25. GPS+Galileo solution convergence after 12 warm starts at 20 km/h.

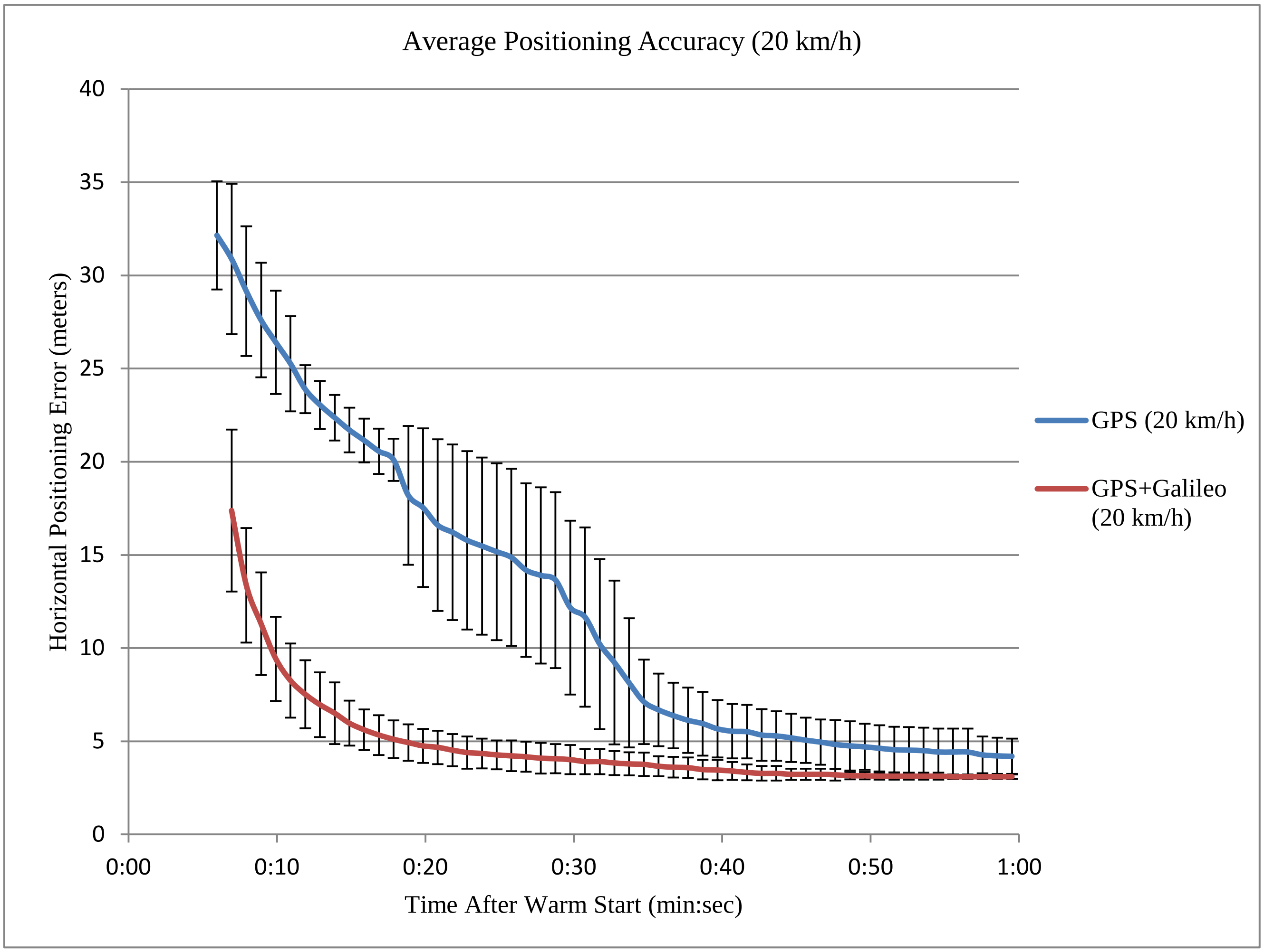

Figure 26. Comparison of average GPS and GPS+Galileo convergence with standard deviation bars.

Overlaying the convergence rates for both static and dynamic test runs in Figure 27 shows comparable receiver performance. Therefore the submarine's low speed would have negligible to no effect on satellite acquisition and TTFGF. For faster submarines, such as the American VIRGINIA class which have a disclosed speed of 25+ knots (46+ km/h), their speed is still relatively slow compared to aircraft and still should not be a factor in satellite acquisition and solution convergence. Note that the submarine's pitch, roll, and yaw are not accurately replicated by a vehicle travelling forward- motion from the sea which could have impact on GNSS receiver performance, particularly in high sea states.

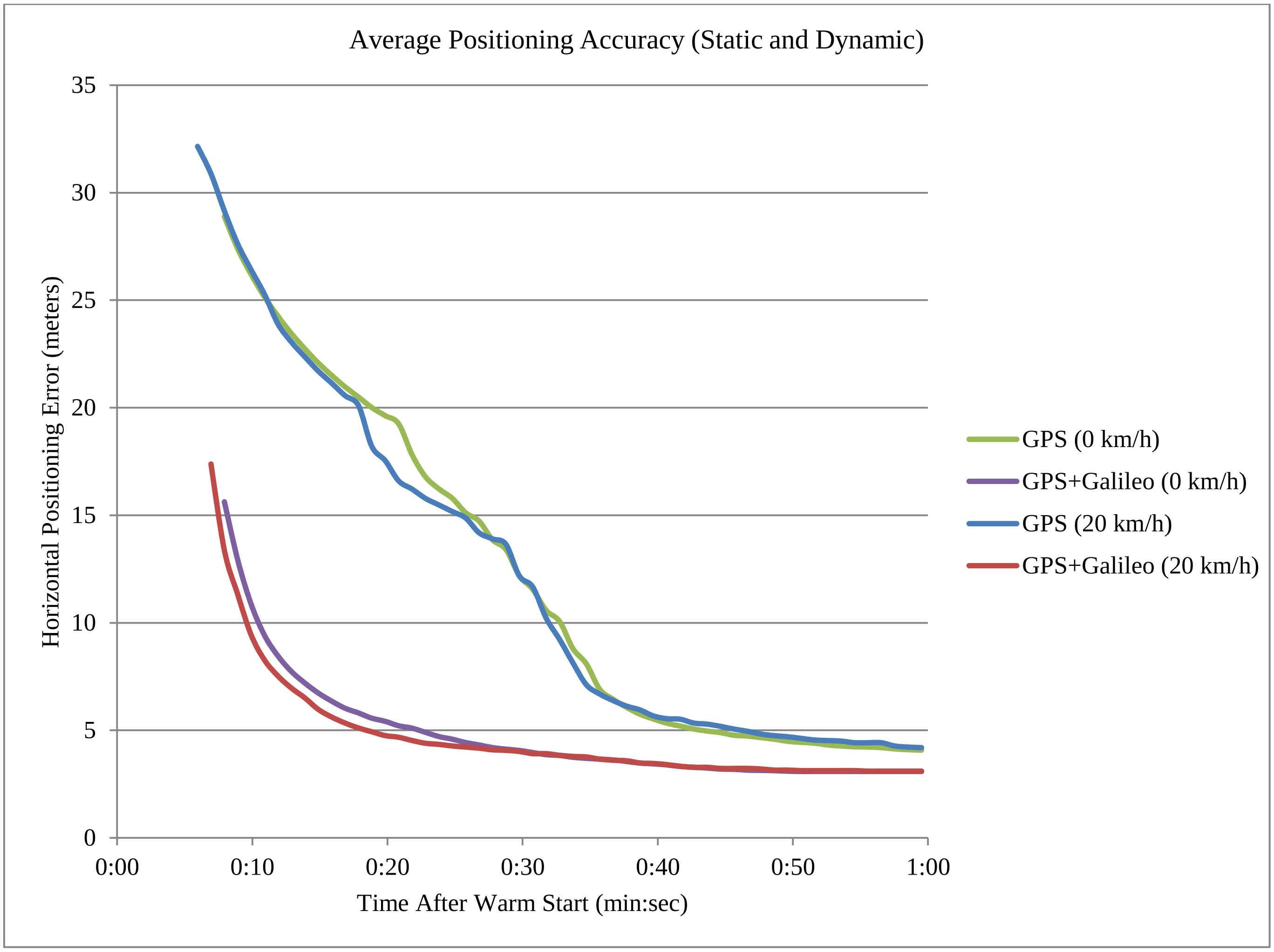

Figure 27. Comparison of average GPS and GPS+Galileo solution convergence under static (0 km/h) and dynamic (20 km/h) conditions.

Chapter 5

Statistical Analysis

This Chapter examines the results presented in Chapter 4 with statistical analysis of each receiver individually. A mathematical comparison of the two receivers follows.

5.1 Analysis of Results

In all results presented below, positioning accuracy readings were logged at a 1 Hz interval while both receivers were initiated in the warm state. The test was run 25 times on different days to ensure varied satellite availability and satellite geometry. Testing over a series of different dates and time of day shows the general performance of GPS-only and GPS+Galileo positioning calculations, with relative consistency across tests showing negligible difference due to time of day and weather conditions. Since GNSS satellites are in constant motion around the planet, and they can thus have an infinite number of positioning combinations relative to the user, it is not expected that identical results could be obtained again. However, trends in the data can highlight operational capabilities that could be exploited by submarines.

Regardless of TTFF, since the submarine needs accurate positioning data, this research is investigating if using the American and European signals could reduce the submarine's total wait time for an accurate EP. Beyond the first good fix, long-term accuracy sustainment is also important for as long as the antenna is exposed out of the water.

Similarly to how this data collection could not be conducted on an operational submarine in open waters, the performance and accuracy threshold of a submarine INS are sensitive in nature and thus not disclosed in available literature. Thus for an INS integrated with the two GNSS receivers selected in Chapter 3, an accuracy threshold is assigned for the assessment of TTFGF values. For a hypothetical submarine integrated GPS/INS paired with either of these two receivers, the accuracy threshold is set as the average positioning accuracy (12 feet) with a 10% buffer (13.2 feet). Since the receivers report positioning accuracy in whole numbers, the positioning accuracy threshold for this study is 13 feet (4 meters).

5.2 GPS Positioning

The GPS-only receiver demonstrated an average TTFF of 3.8 seconds, ranging between 2 and 8 seconds. With the receiver already booted when the warm start is initiated, the 3.8 seconds covers $T_{acquire}$, $T_{bit\text{-}synchronization}$, and $T_{compute}$. However, the average positioning accuracy of that first fix was 37.9 meters- far above the 4 meter accuracy threshold of this paper's hypothetical GPS/INS.

With NVM and enough channels to acquire every satellite in view, there is no longer a requirement to fully download the 12.5 minute almanac data and then conduct a full sky search prior to calculating a position estimate – a significant difference from the first GPS receivers that only operated with five channels and required a fresh almanac download upon initialization. Although the ephemeris and clock data is transmitted over a span of 30 seconds, TTFF values and convergence within the first 30 seconds after the warm start shows that positioning is able to be calculated as ephemeris and clock data are still being received; however, the fix will not be accurate until that whole navigation data message is received and decoded.

Figure 21 shows a noticeable refinement in positioning accuracy immediately following the 30-second mark in numerous test runs. For a submarine either at PD or surfacing in open waters, like the open field conditions on this data collection, the lack of obstructions to interfere with satellite acquisition means the submarine is most likely to download the ephemeris and clock data on the first 30-second cycle possible and calculate a position estimate. If at any point in the first 30 seconds the data download is interrupted by a rogue wave, the receiver will wait for the subsequent 30-second cycle to download any data that was missed. The receiver will then use the subsequent time to refine its position estimate with as many satellites as the receiver will allow.

Figure 28 demonstrates the cumulative distribution of the GPS TTFGF values. A mean time of 59.8 seconds and median time 59 seconds was required to converge to the accuracy threshold of 4 meters. The minimum time to achieve the accuracy threshold was 33 seconds (0:33) and the maximum time required was 111 seconds (1:51). The 90 percentile value was 81 seconds.

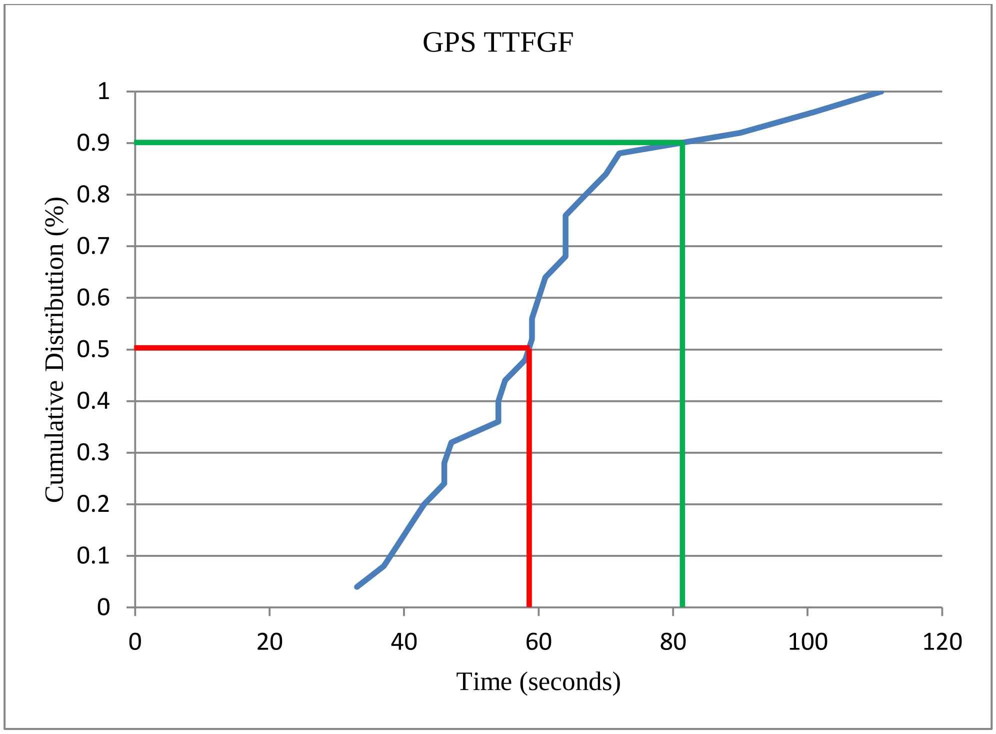

Figure 28. Cumulative Distribution of the GPS TTFGF values. The 50th and 90th percentile values are 59 and 81 seconds, respectively.

Interestingly, all test runs show high convergence rates between the 18-second and 36-second marks after the warm start, which is highlighted in Figure 29. Anghileri *et al*. (2008) states that a receiver can catch the GPS broadcast and start downloading data at any point within its broadcast cycle; thus the reading start point can be considered a random variable uniformly distributed over the 30-second frame. However, since only the ephemeris and clock data (which are covered in the first 18 seconds) are required for a position estimate, the high convergence rate starts at the 21 second mark – 2 seconds is taken for $T_{acquire}$ and 1 second $T_{bit\text{-}synchronization}$.

Since the warm start can be initiated at any point in the broadcast of navigation data, there is randomness in where the reading start point is located through the 30-second message. Thus the time that the receiver has fully downloaded sub-frame 1-3 is a random variable between 21 seconds and 39 seconds (3+18+18 seconds) as seen in Figure 29. The frequency of occurrences for the high convergence rate is plotted in Figure 30. Only with more testing could it be confirmed if convergence rates are uniformly distributed over that time span.

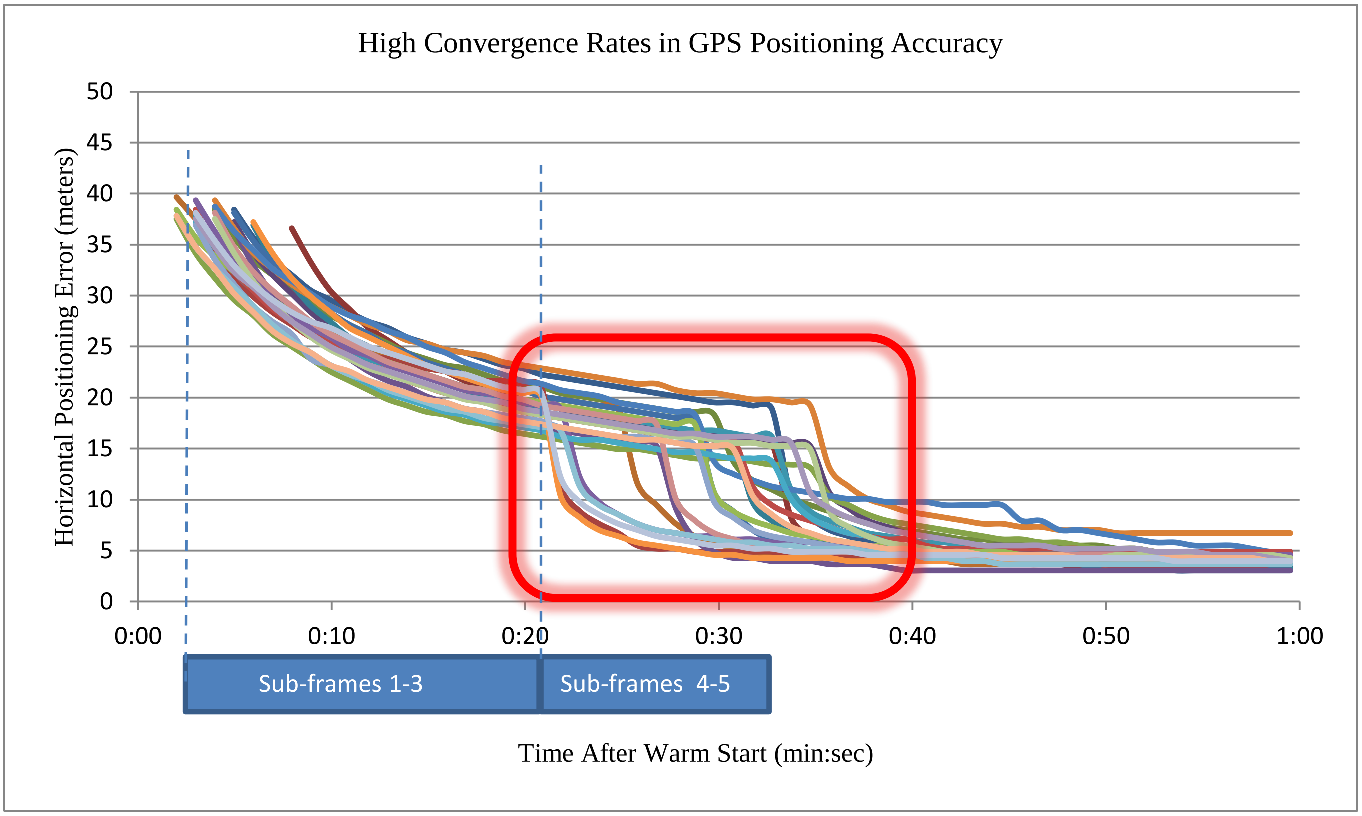

Figure 29. GPS positioning has high convergence rates upon complete receipt of sub-frames 1-3 of the navigation data message.

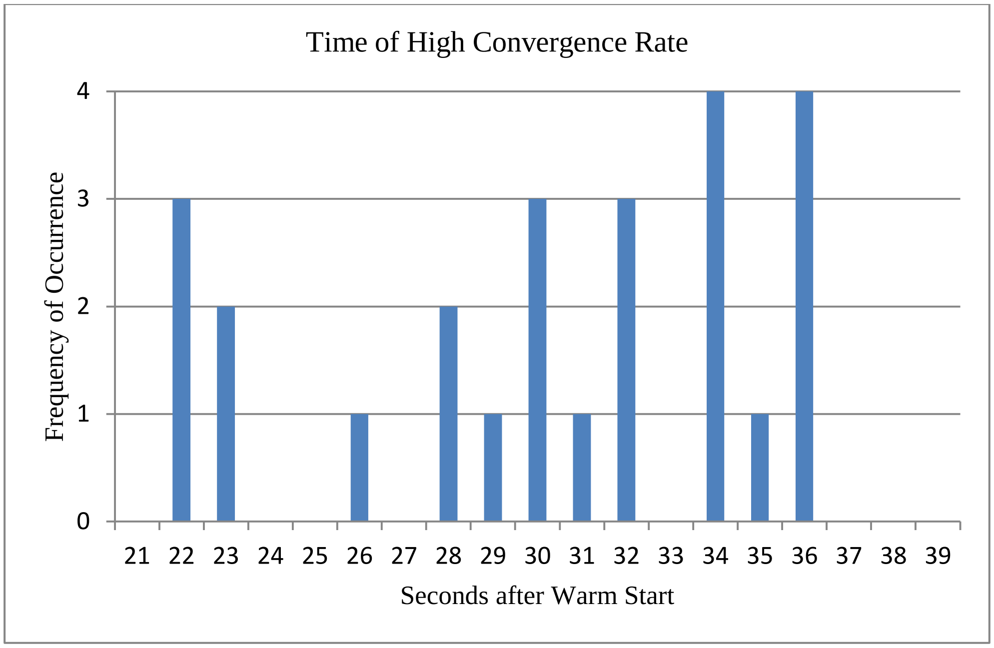

Figure 30. The frequency of occurrence of high convergence rates between the 21 and 39 second marks after the warm starts.

After reaching the accuracy threshold and being left on with open sky conditions, the GPS-only receiver demonstrated an average positioning accuracy of 3.5 meters although with some variation that remained within the accuracy threshold (≤ 4 meters). On some tests though, the GPS positioning accuracy degraded to the point where it exceeded the accuracy threshold, as seen in Figure 31, although only marginally. The degradation of positioning accuracy could be the result of GNSS errors or worsened DOP. Although the positioning accuracy remained less than 5 meters, the consequence is that those position measurements would not be usable by the INS until the positioning accuracy converged again.

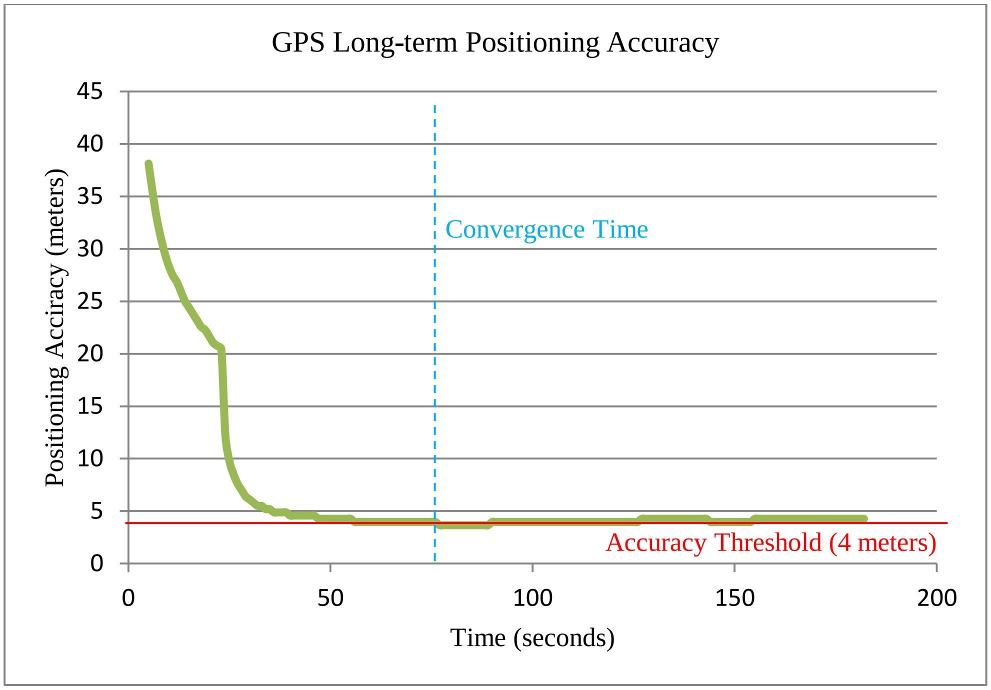

Figure 31. On some test runs like this one, the positioning accuracy exceeded the accuracy threshold of 4 meters after the convergence time, although only marginally.

5.3 GPS+Galileo Positioning

The GPS+Galileo receiver demonstrated an average TTFF of 6.4 seconds, with an average positioning accuracy of that first fix being 24.0 meters.

The cumulative distribution of the GPS+Galileo receiver TTFGF is seen in Figure 32. The GPS+Galileo receiver achieved the 4-meter accuracy threshold within a mean time of 29.2 seconds and median time of 31 seconds after initialization. The minimum time required to achieve the accuracy threshold was 14 seconds and the maximum time required was 51 seconds. The 90 percentile value was 41 seconds. As a newer constellation with a more modern navigation data message, Galileo was designed with low fixing times in mind. Thus the position estimates that met the accuracy threshold in less than 30 seconds are most likely a result of at least four Galileo satellites being available at the time of receiver initialization.

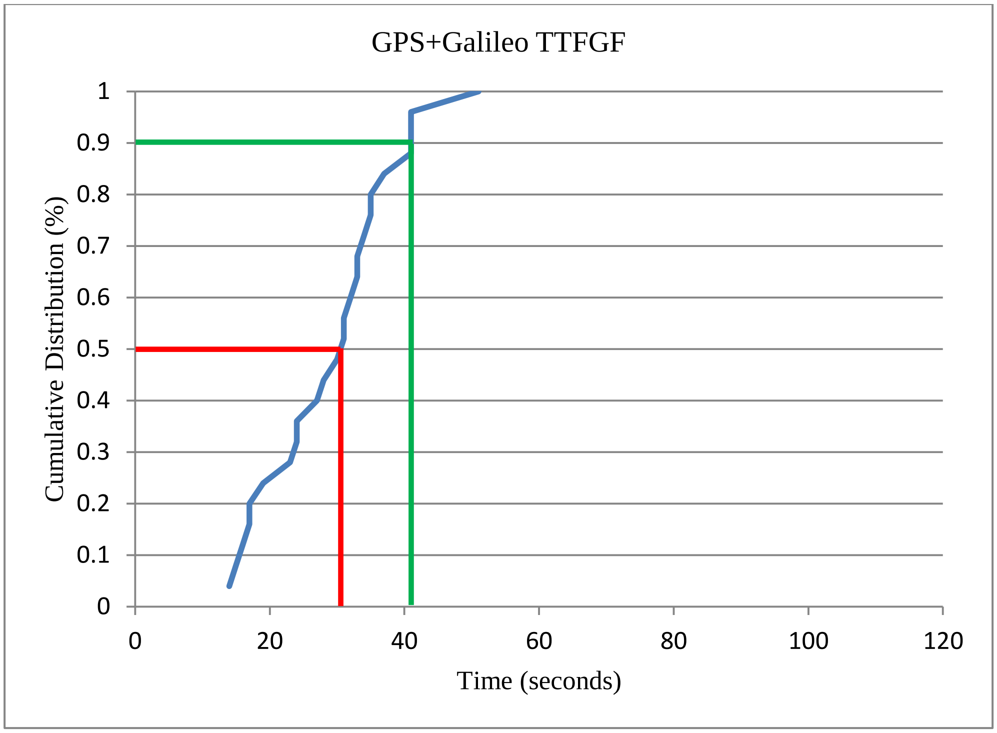

Figure 32. Cumulative Distribution of the GPS+Galileo TTFGF values. The 50th and 90th percentile values are 31 and 41 seconds, respectively.

In terms of solution convergence and the TTFGF, the GPS+Galileo receiver demonstrated that it is not as dependent on the structure and timeframe of the GPS message. Of the twenty-five test runs with Galileo integrated with GPS, the receiver was able to output positioning measurements that met the accuracy threshold in less than 30 seconds eleven times, or 44 percent of the time.

With more satellites available and more optimal DOP, the GPS+Galileo receiver had a superior average positioning accuracy of 3.05 meters. In all the test runs, the GPS+Galileo receiver's positioning accuracy never degraded after it converged, remaining 3.05 meters for the entirety of the test runs.

5.4 Receiver Comparison

The TTFF for both receivers is relatively consistent throughout the 25 tests. In every test with the receivers side-by-side, the GPS-only receiver always had a fix first, which contradicts literature that said that one benefit of multi-GNSS receivers is their lower TTFF compared to single-GNSS receivers (Bowler & Wall, 2014; Cao, 2008; Cozzens, 2023a; Pan, 2014). The GPS-only TTFF averaged 3.8 seconds and ranged between 2 and 8 seconds, while the GPS+Galileo TTFF averaged 6.4 seconds and ranged between 5 and 8 seconds. The frequency of TTFF values for both the GPS and GPS+Galileo receivers results in rough normal distributions as seen in Figure 33. GPS TTFF values follow a right-skewed distribution while the GPS+Galileo TTFF values are more normally distributed. A normal distribution would be expected to form upon the completion of more testing since the factors that determine TTFF – having at least four satellites in view, the message structure, receiver architecture, and receiver state (warm starts) - are all consistent between tests. The larger TTFF of the multi-GNSS receiver can be attributed to the additional time-synchronization, decoding, and demodulation required when dealing with two different systems (the GGTO) and navigation message structures.

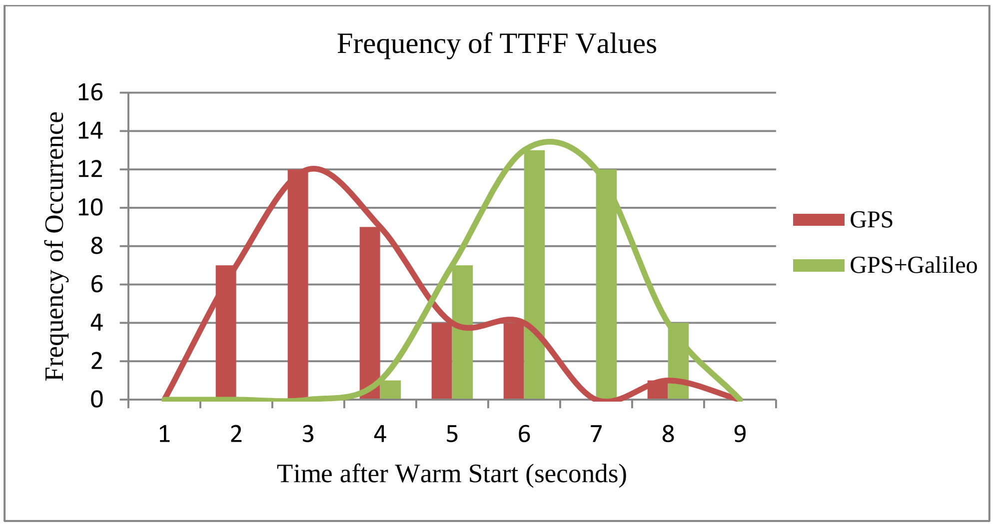

Figure 33. Distribution of TTFF values for both the GPS and GPS+Galileo receivers.

The data messages arriving from GPS and Galileo satellites are affected differently by noise due to the difference in relative power distribution between the data and pilot channels (Pena *et al*., 2009). The Galileo data is arriving at the receiver with twice the power as the GPS data. With a higher power for the signals compared to noise, each Galileo signal should have a higher C/N_0 which is beneficial for the demodulation process. However, the GPS messages have 50% more power allocated to the Pilot Channel than Galileo does, resulting in better tracking performance than with Galileo signals.[28] With the exception of the time

[28] The higher power assigned to the pilot channel in the GPS message benefits receivers in motion although they are more susceptible to noise.

required to synchronize the signals with the receiver's internal clock, the GPS and Galileo data messages are processed simultaneously as they are received. There is no preference or priority of one constellation's signals over the other, and the positioning estimate and convergence rate are functions of all visible satellites.

Small differences in signal transmission and the data message structures can cause small differences in signal acquisition and TTFF. For example, GPS ephemeris and clock data are located in sub-frame 2 while being held in Galileo's words 1 to 4 (Pena *et al*., 2009); however, since the receiver can be initiated and connect at any point in the data message transmission, the location of key data within transmissions is negligible in average solution convergence over multiple test runs. Another discrepancy that may cause minute differences between GPS and Galileo TTFF is the repetition of data. While the broadcast cycle of GPS ephemeris and clock data are constant every 30 seconds, the Galileo reserve bits cause its ephemeris and clock data to be variable with every transmission (Pena *et al*., 2009). Individually, GPS and Galileo offer similar TTFF, positioning performance, and convergence rates. While the discussed differences in signals and message structure may cause small differences in satellite acquisition, TTFF, and positioning performance, they are not significant enough to impact convergence rates over the span of minutes.

The different TTFGF values, not only between tests but also between the two receivers, demonstrate how solution convergence and TTFGF are more dependent on factors besides the GNSS signal strength, frequency, and data message structure. Positioning accuracy and solution convergence are more dependent on satellite availability and satellite geometry, as outlined in Figure 13. Unlike on land where GNSS users are usually surrounded by obstacles, the submarine's full FOV means the satellites with the highest elevation are not necessarily the satellites to be used for positioning; instead, the receiver has approximately double the number of interoperable satellites to work with including those at lower elevation for better HDOP as seen in Figure 19, causing a higher convergence rate compared to the GPS-only receiver. Furthermore, the variety of frequencies assists in error correction, primarily ionospheric delay, to refine the solution, leading to a quicker convergence. If the additional satellites did not play a role, the TTFGF of the two receivers would be similar throughout the twenty-five test runs.

Unlike TTFF, the distribution of TTFGF does not follow a normal distribution as seen in Figure 34. TTFGF depends on satellite DOP which is not consistent between tests. Also not consistent between tests is the reading start point. The receiver can be initiated and catch the satellite broadcast at any point within its broadcast cycle, making it a random variable over the span of the message broadcast. If all required navigation data is received on the first possible broadcast, it is expected that the TTFGF values is univariate in that the variable (TTFGF) takes on a number of different values.

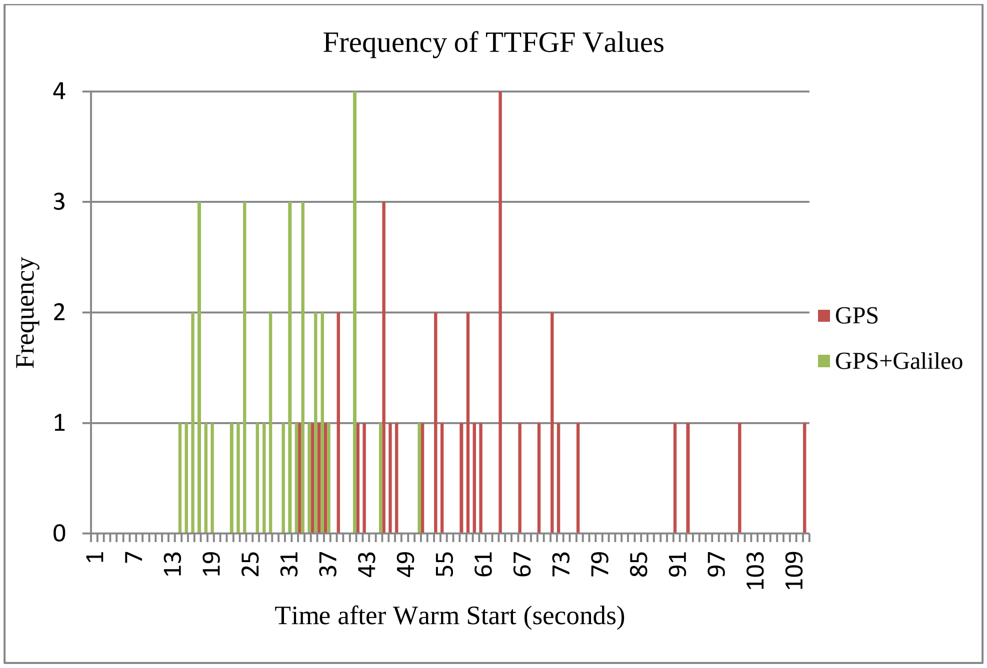

Figure 34. Univariate distribution of TTFGF values for both the GPS and GPS+Galileo receivers.

In open sky conditions, like the conditions that the submarine would experience, a combination of GPS and Galileo satellites will always make more satellites available to the receiver than working with just one constellation. This would also apply in GNSS-challenged environment, such as urban centers where buildings obstruct direct LOS with many satellites. In the long run after solution convergence, the fact that the two systems were designed to be interchangeable also increases positioning accuracy and system reliability. However, the abundance of pseudoranges means more satellite signals arriving at the receiver can have errors that can degrade positioning accuracy (Siemuri *et al*., 2022). In order to capitalize on high satellite availability, the receiver must select and range off the optimal visible satellites by minimizing DOP on its own.

With more available satellites and more optimal DOP at all times, the GPS+Galileo solution converges at a higher rate than the GPS-only receiver. The median wait time for a good fix was shortened from 59 seconds to 31 seconds by integrating Galileo signals into the computation, saving 28 seconds. The kinetic test runs exhibited similar values, with the integration of Galileo saving 27 seconds from the wait time for a submarine traveling at its maximum surface speed. With the similar convergence rates of the receivers under static and dynamic conditions (Figure 27), combining the results together gives a median GPS-only TTFGF of 60 seconds and median GPS+Galileo TTFGF of 33 seconds. While there is variation in the TTFGF times of the two receivers, the amount of time saved in waiting for a first "good fix" by integrating Galileo is more consistent. These results are summarized in Table 10.

Table 10. Summary of the GPS and GPS+Galileo receiver performance.

Receiver Speed	GPS Median TTFGF	GPS+Galileo Median TTFGF	Reduction in TTFGF with the Integration of Galileo	Percentage of GPS-only TTFGF Saved
0 km/h	59 seconds	31 seconds	- 28 seconds	47%
20 km/h	64 seconds	37 seconds	- 27 seconds	42%
All	60 seconds	33 seconds	- 27 seconds	45%

The GPS+Galileo receiver was able to output positioning data that met the GPS/INS accuracy threshold in less than 30 seconds eleven times (44% of the time). The GPS-only receiver was never able to output accurate positioning data in less than 30 seconds, requiring at least 33 seconds to meet the GPS/INS accuracy threshold.

Upon the solutions converging, not only did the GPS+Galileo receiver demonstrate superior positioning accuracy than the GPS-only receiver due to the increased satellite availability and better DOP (Trautvetter, 2018), but it also demonstrated better positioning stability in that it never wavered above 3.05 meters after converging. On the submarine, such stable positioning data by the multi-GNSS receiver would help refine the integrated GPS/INS solution by enabling more stable tuning (Cahyadi, Asfihani, Madriyanto, & Erfianti, 2022).

While this research has shown that a GPS+Galileo receiver exhibits lower TTFGF than GPS-only receivers, it would be subject to minor drawbacks along with other multi-GNSS receivers. Foremost, the GPS+Galileo receiver is more costly due to the additional technology and complexity in tracking multiple constellation signals simultaneously. Additionally, although more of a concern for handheld receivers with limited battery life, tracking additional signals (particularly those of other systems) also demands more power.

5.5 Analysis Summary

This research shows that a GPS+Galileo receiver solution converges quicker than a GPS-only receiver solution under the same conditions. If equipped on a submarine, the multi-GNSS receiver would have a shorter TTFGF than a single-GNSS receiver, resulting in less exposure time of the antenna when at the surface of the water. The expedited solution convergence is primarily due to greater satellite availability and improved satellite geometry from the integration of Galileo. The subtle differences between the GPS and Galileo data message structure and data rates are not substantial enough to impact convergence rates or TTFGF. Although a first fix is computed slower by the GPS+Galileo receiver, having more satellites available and greater satellite geometry diversity enables the receiver to deliver a more accurate first fix and achieve a higher convergence rate. In the long-term, the increased number of available satellites also exhibits greater positioning solution stability with less variance from superior error detection and signal redundancy.

However, these results do not exactly reflect how they would appear on a submarine for two main reasons. First, the military employs dual-frequency encrypted receivers, which both ensure signal security and improve positioning performance – the latter reducing TTFGF more than that seen in this study. Second, the submarine's GNSS receiver is not standalone, as discussed in Chapter 2. The integrated INS provides the submarine's approximate location and velocity to the GPS receiver via acquisition aiding specifically to reduce its search space. This data helps the code generator and oscillator make better-informed estimates on incoming signals' frequencies and code phases. When paired with the almanac data stored in NVM (if still valid), the receiver can avoid looking for satellites that are out of its LOS. The result of the feedback data is a reduction in $T_{acquire}$ and thus both TTFF and TTFGF. Both the use of encrypted receivers and an integrated GPS/INS would result in abbreviated TTFGF values for both receivers, but it is still expected that the combined GPS+Galileo receiver would out-perform the GPS-only receiver in convergence to the accuracy threshold.

The accuracy threshold used in this study was 13 feet / 4 meters, which was based on the long-term maximum observed positioning accuracy of the GPS receiver, with an additional 10% buffer. The 4-meters accuracy threshold is a "worst case" since both receivers take the most amount of time to reduce their positioning error from five meters to four meters than any other 1-meter increment. In general, the accuracy threshold and the time required for the receiver to reach that positioning accuracy are inversely proportional. Figure 35 shows that if the GPS/INS accuracy threshold limit were eased and set higher, less time would be required for positioning data to meet the accuracy threshold from a warm start.

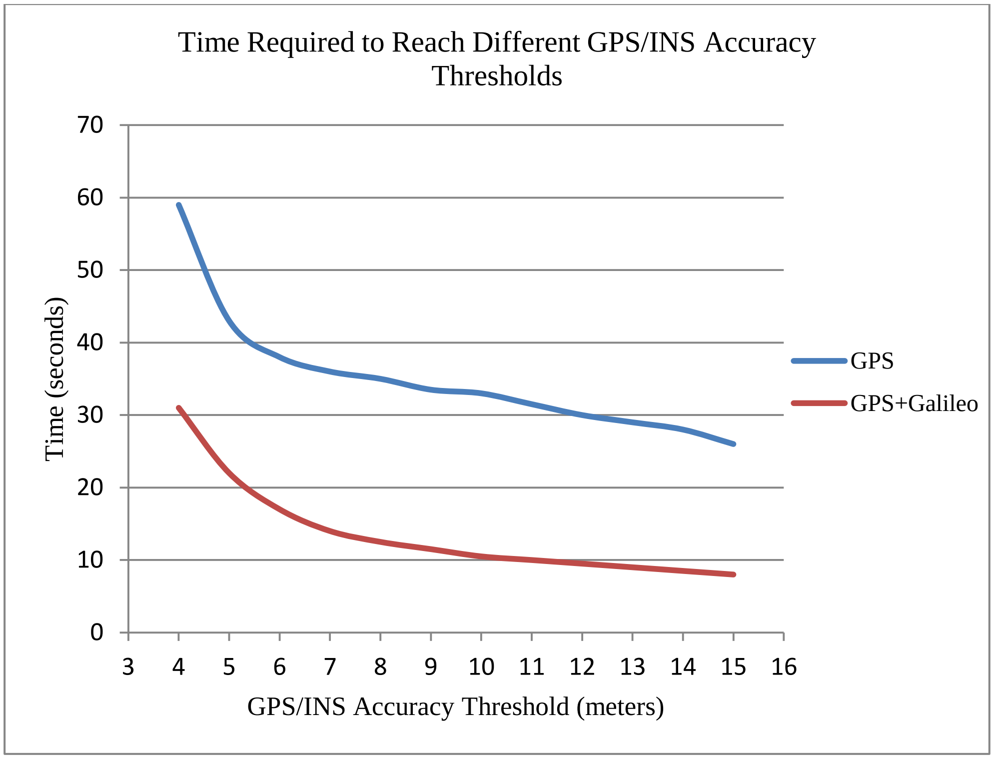

Figure 35. Time required to reach different accuracy thresholds on the two receivers.

By analyzing the stationary convergence rates in Figure 23, the amount of time saved by using a combined GPS+Galileo receiver generally decreases as the accuracy threshold is increased. Between a variety of accuracy threshold values of 4 meters and 15 meters (the maximum, since 15 meters was the average positioning accuracy at first fix), the amount of time saved by using the combined GPS+Galileo receiver generally decreases. Although the GPS-only receiver always output a first fix quicker than the GPS+Galileo receiver, the latter always output more accurate positioning data and thus meets all possible accuracy thresholds first. The amount of time saved in reaching the accuracy threshold by integrating Galileo instead of using a GPS-only receiver is illustrated in Figure 36. The average time saved by using a GPS+Galileo receiver is 21.5 seconds, saving between 47% (4-meter accuracy threshold) and 69% (15-meter accuracy threshold) of time that would be needed for the GPS-only to reach that same positioning accuracy.

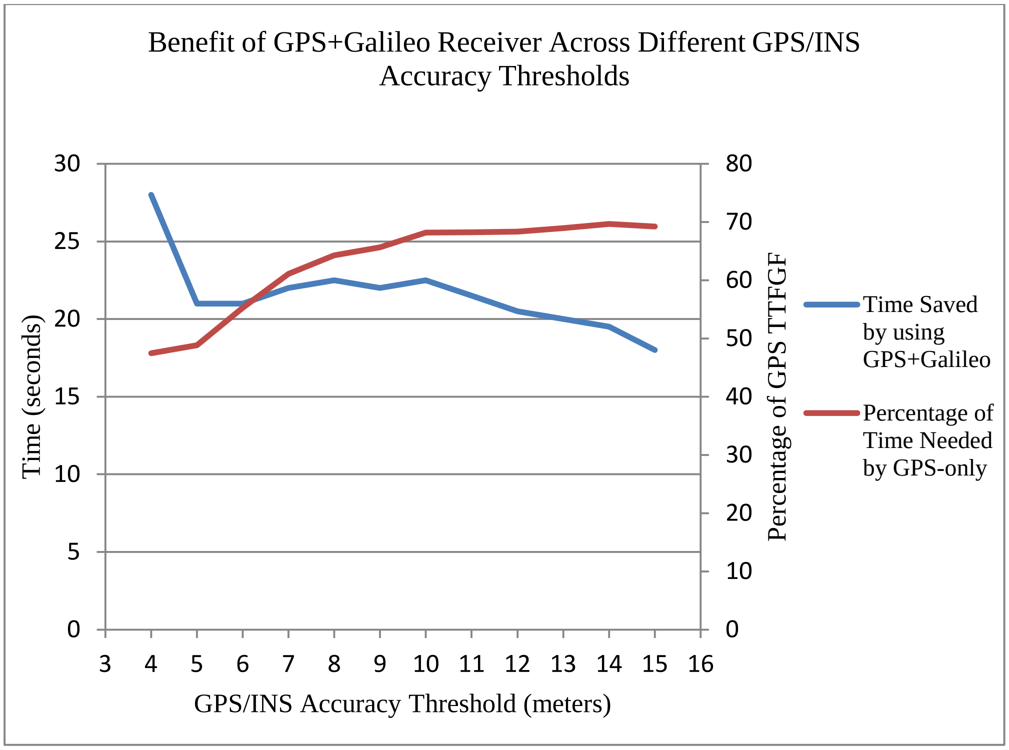

Figure 36. The amount of time saved in reaching various GPS/INS accuracy thresholds by integrating Galileo.

Although the GPS+Galileo receiver save a greater percentage of time compared to the GPS-only receiver in waiting for accurate positioning data with higher accuracy thresholds (≥10 meters), those higher accuracy thresholds correlate to a less accurate GPS/INS navigation system. Selecting a higher accuracy threshold and then processing less accurate positioning data into the submarine's GPS/INS will result in faster POE growth. The submarine's navigation system, along with other complex navigation systems, would keep to the lower end of the accuracy threshold spectrum, despite the GPS+Galileo receiver needing less of a percentage of the GPS-only receiver's TTFGF for those lower accuracy thresholds.

Note that the results presented in this research are particular to the Garmin receivers and the GPS and Galileo systems at the time of testing. As GNSS systems evolve to remain relevant and increasingly accurate through the twenty-first century, their orbital configuration, signal, and navigation data messages may all be changed to better the user experience. Likewise, receivers will continue to evolve and improve their satellite acquisition strategies, signal processing methods, and error detection and correction techniques.

Chapter 6

Discussion

This Chapter discusses the relevance of the results of this thesis. Not only do the results pertain to submarine positioning, but they also pertain to the wider GNSS community – albeit in a small capacity. The discussion chapter will also address the implementation of GPS+Galileo capability on submarines and addresses other options to reduce submarine TTFGF that could be addressed in future research.

6.1 Application of the Results

Not too many GNSS users are as passionate about low wait times to achieve an accuracy threshold as those navigating submarines. The majority of GNSS users can either leave their receivers permanently on or can afford to wait the few minutes for the solution to converge for accurate positioning data. A submarine however, needs accurate positioning data as soon as possible following the antenna breaching the water surface. These results are applicable to both the current VICTORIA Class submarines and future submarines to be commissioned through the Canadian Patrol Submarine Programme (CPSP). The results show that a combined GPS+Galileo antenna offer submarines a lower wait time between receiver initialization and the first good fix, shaving the wait time for an accurate fix by 47%. These results are also applicable to the greater GNSS community in other capacities.

6.1.1 Application to Submarines

While previous research has shown that multi-GNSS receivers are more accurate over time, this research shows a novel benefit to a submarine- less wait time between receiver initialization and positioning data reaching the GPS/INS accuracy threshold. A shorter TTFGF means the submarine's GNSS antenna does not have to be exposed above the water surface for as long as a GPS-only antenna (47% less time), limiting the submarine's exposure to the surroundings where it can be detected visually, by RADAR, or other remote sensing capabilities. The less time a submarine remains exposed at the surface, or near the surface for that matter, the less chances it has of being detected. Rapid fixing is most important for inshore/coastal operations, where the submarine has limited time on any course and speed to remain exposed on the surface, particularly in congested waters. A lower TTFGF also means less distance travelled in the time it takes to get an accurate EP, which is particularly important if visibility is poor.

If paired with a 360° periscope camera, which can digitally capture the surroundings in an instant rather than be turned for a submariner's manual observation, the periscope and equipped GNSS antenna could be raised for a fraction of the time it currently requires for a position fix, as well as for assessing the submarine's surroundings. After the fix is obtained within the median time of 33 seconds, the periscope could be retracted and the panoramic digital picture analyzed from below the water surface, leaving the submarine just below the surface with no mast to compromise its position until the next visual assessment is required. The less time the periscope is exposed, the less chance there is of counter-detection.

In addition to quicker solution convergence, the positioning accuracy of the GPS+Galileo receiver was less varied, never faltering above the accuracy threshold unlike the GPS-only receiver which has a certain level of uncertainty. Stable positioning is a result of the additional satellites, but also from the absence of multipath effect in open waters; the more significant source of error for the submarine is atmospherics, whose effects are mitigated by the use of a dual-frequency receiver (Boguspayev *et al*., 2023). Stable positioning data via the multi-GNSS receiver will help refine the integrated GPS/INS solution by enabling more stable tuning with less accuracy violation alerts (Cahyadi, Asfihani, Madriyanto, & Erfianti, 2022; Rahman, 2022). When paired with an integrated GPS/INS, in a loosely-coupled architecture, the receiver's Kalman filter will have the responsibility of maintaining that confidence level in the GPS position measurements. Measurements with low uncertainty can be processed to have greater impact on the system state.

With improved positioning data plugged into the submarine's INS, equipped with either RLG or newer fiber-optic gyro (FOG) systems, submarines will be able to navigate more precisely and thus more safely. However, GNSS positioning accuracy and fixing times are not related to the POE's growth which force the submarine to the surface for INS position updates. Improvement in INS technologies will help reduce POE growth rates, which when coupled with more precise GNSS positioning, will result in fewer instances at/near the surface where the submarine is more vulnerable. It is unlikely that R&D develops an INS that does not require external (satellite) input at all, particularly for moving vehicles like submarines (Goward, 2022).

A GPS+Galileo receiver would give the submarine crew new redundancy in their satellite navigation capability. If signals from either GNSS were degraded or compromised altogether, the submarine would still be able to calculate PNT solutions from the other GNSS. The vulnerability associated with relying on a single GNSS was evident in July 2019, when for eight days the PNT aspect of the Galileo constellation was unusable due to a technical difficulty with orbit and time predictions within the ground infrastructure (Amos, 2019; EUSPA, 2019). Throughout that time, users were encouraged to rely on the other GNSS satellites for all positioning and time needs, if their devices could. Although Galileo was still in its pilot phase at the time (Amos, 2019), the outage highlighted the importance of being able to work with multiple systems.

Regarding the submarine's navigation system, integrating Galileo would result in no change to redundancy and survivability. Following implementation of a GPS+Galileo antenna and receiver, there would likely be the same number of antennas and receivers onboard, with the identical interfaces (namely power and navigation data output) as the currently-fitted GPS components. As for survivability, the layout and physical location of all necessary parts would be unaffected.

In terms of system security, recall that convergence and positioning accuracy are affected by signal interference. The low power of all GNSS signals makes the system susceptible to both intentional and unintentional electromagnetic interference (Marvel, 1998). However, military vehicles including submarines exclusively use PPS which provide greater immunity from attack.[29] The additional frequencies when integrating Galileo provide further system security since jamming multiple frequencies simultaneously is more complex. In the event the submarine finds itself in a complete GNSS-denied environment, it will place greater reliance on its INS for all positioning data.

Aside from initial cost and implementation requirements, there are no significant drawbacks to applying multi-GNSS receivers on submarines. The increased power demand of multi-GNSS capability compared to GPS-only receivers is negligible due to the submarine's own supply of electrical power. Altogether, the implementation of a GPS+Galileo receiver onboard a submarine would deliver significant benefits in terms of TTFGF and positioning accuracy at low cost and low complexity.

6.1.1.1 Implementing GPS+Galileo Receivers on Submarines

Particularly with Canada's strong partnership and alliances with the EU, Canada should take advantage of the readily-available, no-cost, and worldwide opportunity that Galileo presents. After GPS, Galileo is the next most likely GNSS to be used by Canada for its military PNT requirements.

While the process for EU member states to acquire and implement Galileo PRS receivers is straightforward, the process for third party nations (like Canada) is more complex. Each country must prove to the EU that they have the necessary security protocols to manage and control PRS receivers (Cozzens, 2021). If

[29] An adversary needs a higher powered jammer to break a PPS receiver's P(Y) code lock (Schmidt, 2015).

approved in a PRS security agreement, the EU will then export the necessary equipment, including the receivers developed under the GEODE program (discussed in Chapter 2), to those countries. Along with the agreement comes an "observer status" to the EU's space program but no decision-making powers or involvement in encrypting PRS signals (Besch, 2018). The U.S. applied for such an agreement in 2021 (Ackermann, 2021; Leonardo, 2021), which if approved, will enable the manufacturing of combined GPS and Galileo receivers for military use.

With current RCN GPS receivers manufactured by Trimble Inc., the most likely course of action would be procurement of a Trimble GPS receiver with Galileo capability. With Trimble being an American company, the FCC's 2018 waiver for Galileo capability would apply, enabling the company to manufacture, test, and operate with Galileo signals. At the time of this writing, Trimble produces multi-GNSS antennas with GPS, GLONASS, and Galileo capability for the commercial market, which could be adapted for use with the decryption keys for military applications. Leonardo is also developing integrated GPS/Galileo receivers that will deliver maximum performance for both civilian and military applications (Leonardo, 2021).

In terms of antenna and receiver installation, distance between the two reduces electrical noise but increases latency, calling for optimized placement of hardware. On the submarine, distance between the two is inherent with the antennas on the masts[30] and the receiver within the pressure hull as part of the navigation data network whose main infrastructure is co-located with the INS. A GPS+Galileo antenna would likely occupy a similar footprint as the currently fitted GPS antenna, as would the new receiver, which is critical for a submarine where space is limited. Also expected to be the same would be the power demand from the submarine and wiring configuration particularly for data output. All associated wiring should be adequately shielded for maximum performance. In order to certify the new receiver, the navy would be required to develop the necessary tests to verify positioning performance - likely mirroring those already in existence to verify GPS performance.

Once installed and verified, a GPS+Galileo receiver would need to be integrated into the submarine's navigation system and tested for compatibility with its existing systems (outlined in Chapter 1) that may have been designed with GPS-only receivers in-mind. To date, there is no available literature highlighting interference issues from users working with both GPS and Galileo. The receiver's performance would then need to be validated in all expected operating conditions and environments, including GNSS interference. The cryptography nature of PRS

[30] Placed as to have adequate distance from any masking structure, such as other masts (Pandele *et al*., 2020).

receivers should mirror those of the GPS PPS receivers, ensuring interoperability of secure systems from dual systems.

6.1.2 Application to the GNSS Community

Submarines are a very small segment of GNSS users; however, the demand/requirement for quick convergence rates is also useful to users who require quick accuracy and whose GNSS connectivity is intermittent. These include vehicle tracking, personal tracking, and marine research. However, the availability of augmentation services means that not all users who benefit from quick convergence require multi-GNSS receivers for that purpose.

Fast convergence is sought throughout the transportation industry, as more vehicles rely on technology for navigation and companies rely on data for fleet management. It is not ideal for operators to wait for accurate positioning data upon turning on a vehicle before they can start moving. Additionally, vehicles will not necessarily have a full FOV, thus reducing the number of visible satellites and potentially extending the fixing time. Vehicles in GNSS-challenging environments like urban centers that suffer regular loss of satellite locks will also demand low re-acquisition times (Rahman, 2022). Particularly for autonomous vehicles and commercial vehicle tracking, low TTFGF is essential when vehicles can travel a great distance in the time its receiver needs to get both a first and a first accurate fix (Loizou, 2020). Furthermore, any anti-theft tracking devices also demand low fixing times to detect when vehicles and/or merchandise has been moved and where it is moving. These requirements within the transportation industry will demand low TTFGF as to not inconvenience users while also demanding low-cost and high-reliability. Like fleet management, individuals also seek high accuracy and re-acquisition times from personal fitness devices. Online forums are saturated with users unhappy with their devices' tracking performance and inquiring on alternatives for their specific sport and in region. Although both GPS and Galileo are more accurate when using dual-frequency, small devices like watches cannot support dual-frequency receivers. This research shows that small devices can benefit from GPS+Galileo (or any combination of GNSS) while still operating with single-frequency receivers. However, these vehicle management and fitness requirements for quick solution convergence are all terrestrial and with high-likelihood, fall within the coverage of augmentation services discussed in this paper such as A-GPS, which significantly reduce fixing and convergence times.

Within the marine industry, whose vessels regularly proceed to sea and operate outside of augmentation services, still do not require quick convergence since their navigation systems are initiated while alongside (in port) and then maintain GNSS connectivity throughout their voyage or operation. With full FOV, it is unlikely that ships experience signal loss at sea. If their GNSS solution is lost however, they should maintain a series of backup options to determine their position, including RADAR and visual triangulation. When they regain GNSS positioning, quick convergence is not necessarily a hard requirement due to the relatively large distances maintained between ships at sea, meaning they can afford the time (likely 1-2 minutes) for their single-GNSS solution to converge without the exploitation of signals of opportunity.

Shipboard helicopters also experience the same extended fixing times as submarines. Being stored within ship hangars when now in use, maritime helicopters are traversed and their electronics only initiated for flight when required, leading to intermittent GNSS connectivity like submarines. Particularly for those embarked on RCN ships, maritime helicopters could benefit from abbreviated TTFGF from GPS+Galileo receivers particularly in times of emergencies, like personnel overboard and vessels in distress.

The number of GNSS users in maritime environments and remote regions who operate outside the coverage of augmentation services and experience intermittent GNSS connectivity and could benefit from quick solution convergence, other than submarines, is extremely limited. One group of individuals who face the consequence of intermittent connectivity in remote maritime regions are marine researchers, who rely on satellite tracking for marine animals. Any tracking device that is affixed to marine animals that spend time both above and below the water surface experiences the same intermittent GNSS coverage as submarines. When those species breach the surface of the water for limited amounts of time, accurate satellite positioning is imperative for correct data collection and resultant discoveries (Griesel, 2006). Success of that tracking device can be considered dependent on the amount of time that species spends at the surface, allowing the device not only to acquire satellites but for the solution to converge (Griesel, 2006). With species like turtles and dolphins only surfacing for a matter of seconds, this research shows that the GNSS positioning data collected at that moment is relatively inaccurate. Unfortunately, researchers have no control over how long different species will remain above the water at any moment, which determines if the positioning solution can converge. The use of “Fastloc-GPS” technology enables positioning data within tens of milliseconds but consequently suffers from poor accuracy in range of tens of meters (Dujon, Lindstrom & Hays, 2014), which can be detrimental to an analysis. For positioning’s sake, if time permits with some species, a better alternative is multi-GNSS receivers as shown by this thesis, which can output accurate positioning data in as little as 14 seconds. In any research initiative, surfacing habits of different species can be balanced with accuracy requirements and convergence times (Griesel, 2006). Dujon, Lindstrom & Hays (2014) also suggests the use of more satellites whenever possible for more accurate positioning and velocity tracking of marine animals, particularly benefiting research into their movement patterns – findings that are confirmed by this research, particularly for remote maritime environments.

In all these cases, users must be aware that accurate positioning information is not immediate. Also, an accurate fix is more valuable than a first fix, despite manufacturers primarily only advertising TTFF values. Receivers should be affixed where they have maximum exposure (largest FOV) to acquire satellites and need time to calculate a first EP. As satellite signals are measured and known errors addressed, the position estimate is refined over time, converging over the span of the first 1-2 minutes after a warm start.

6.2 Extrapolation of Results

The results presented above can be extrapolated to further the discussion of TTFGF and solution convergence in other circumstances, such as location and with other types of multi-GNSS receivers.

6.2.1 Global Results

Without conducting the same tests in different locations, the results presented above can be extrapolated to determine if TTFGF would be different around the world. Particularly with the anticipated impacts from climate change, the capability for ships and submarines to have full navigational and operational functionality in the Arctic will factor into Canadian sovereignty operations.

Both the GPS and Galileo constellations were designed so that a user anywhere in the world has at least six satellites in view, and thus a submarine operating a multi-GNSS receiver is assured to have not only six but an excess of available satellites in view anywhere on Earth. With both TTFF and TTFGF dependent on satellite availability, conclusions on worldwide TTFF and TTFGF can be formed.

With TTFF a function of satellite availability, message structure, receiver architecture, and receiver state, TTFF is expected to be constant anywhere in the world. For a surfacing submarine, the navigation data messages, receiver architecture, and receiver state (cold state) are all unchanged from the results presented in this thesis. The only aspect that changes for a submarine in the Arctic is the receiver's perspective of GNSS satellites. However, the average number of visible GPS+Galileo satellites from the North Pole is actually higher than the average number at the Equator or mid-level latitudes. Thus the minimum four satellites are always visible regardless of latitude or longitude, meaning that satellite acquisition and TTFF are not either slowed down or hindered by the user's location.

With regards to TTFGF and solution convergence, which are dependent on satellite availability, satellite geometry, and error correction, TTFGF is expected to be constant across all latitudes with GPS and Galileo satellites directly overhead

(55°S ≤ receiver's latitude ≤ 55°N) and counter-intuitively may even be lower at higher latitudes where GNSS satellites are not overhead. At higher latitudes (between 60° and 90°), despite having no GNSS satellites directly overhead, the number of satellites in-view of a receiver actually increases due to their high orbital altitude. With respect to DOP, Hunt (2020) and Wang (2006) show that DOP is a function of latitude due to the orbital inclination of GNSS satellites. As a GNSS user moves towards the poles and satellites appear lower on the horizon, HDOP actually decreases which is better for two-dimensional positioning while GDOP, PDOP, and TDOP all increase (Hunt, 2020). Over the 12-hour period illustrated in Figure 37, GPS+Galileo receivers see an average 81% increase in satellite availability and a 29% reduction in HDOP compared to GPS-only receivers. Since satellite availability and horizontal positioning accuracy both improve beyond 55° latitude, it is expected that TTFGF also improves at high latitude. For these reasons, in addition to dual-frequency receivers correcting for the increased atmospheric error as satellites appear lower on the horizon, positioning solution and TTFGF may actually be quicker for a submarine in the Arctic than the results seen in this thesis.

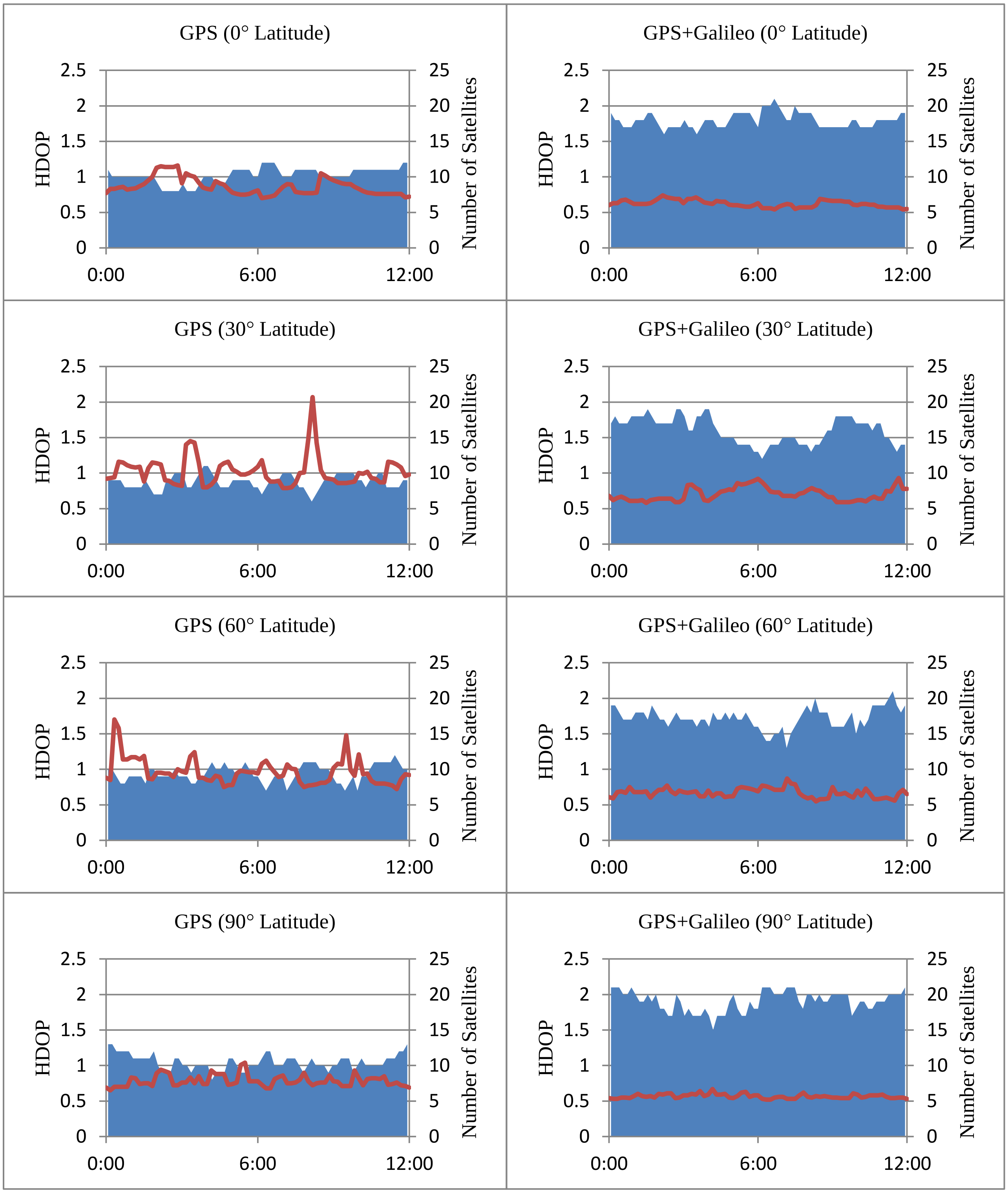

Figure 37. Comparison of HDOP and the number of satellites visible at different latitudes over a twelve hour period. The red line indicates HDOP and the blue area indicates the number of visible satellites.

6.2.2 Other Multi-GNSS Receivers

This research confirms that both positioning accuracy and solution convergence rates improve through the integration of an additional GNSS constellation. It can therefore be expected that receivers compatible with GLONASS and BeiDou, as well as other worldwide signals of opportunity (SBAS for example), would similarly exhibit an increased number of available satellites and improved satellite geometry, which together expedite solution convergence. Additional benefits would also include increased signal accessibility and reduced chance of signal interruption, which helps keep fixing times low.

Garmin, whose receivers were used in this study, also manufactures receivers compatible with GLONASS. The benefit of GLONASS arises in GNSS-challenging environments, like mountainous terrain and high latitudes, since the system provides more high-elevation satellites with more uniform distribution, which is not seen with GPS (Rychlicki, Kasprzyk & Rosinski, 2020). In terms of BeiDou, both GPS and Galileo signal frequencies overlap with the BeiDou B2 signal, facilitating receiver interoperability particularly in multipath situations (Luo, Chen & Richter, 2017). Studies including Luo, Chen & Richter (2017) and Rychlicki, Kasprzyk & Rosinski (2020) saw smaller TTFF and smaller ranges of positioning error when integrating all four GNSS together in a single receiver due increased satellite availability and differences in orbital configurations. However, these more complex receivers, in addition to drawing more electrical power to support their operation, require sensor hybridization and new data fusion techniques while manufacturers must eliminate any signal interference to could compromise receiver performance.

Over a 24-hour period of simulations for a receiver outside of Halifax harbour, a GPS receiver with a 10 degree cut-off had between 6 and 11 GPS satellites within its FOV at any one time. With the same conditions, a GPS+Galileo receiver could consistently see between 11 and 18 satellites, while a receiver compatible with GPS, Galileo, GLONASS, and BeiDou consistently had between 27 and 36 satellites within its FOV (Figure 38). In terms of satellite positioning over the same time period, HDOP improves as more interoperable GNSS are integrated (Figure 39). As both satellite availability and HDOP improve with the further integration of additional GNSS, the quicker the solution will converge and the more accurate the positioning data will be.

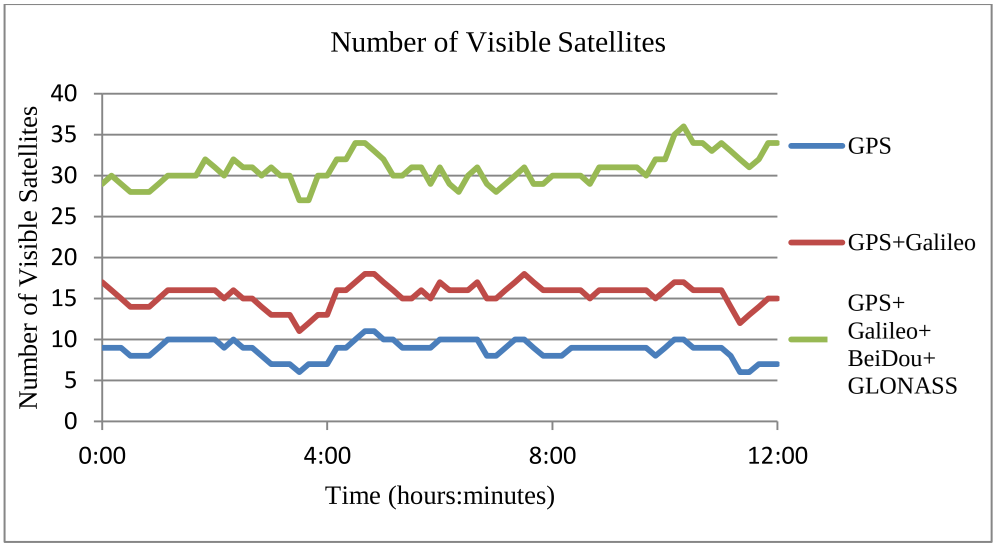

Figure 38. The number of available satellites over a 12-hour span with different multi-GNSS receivers.

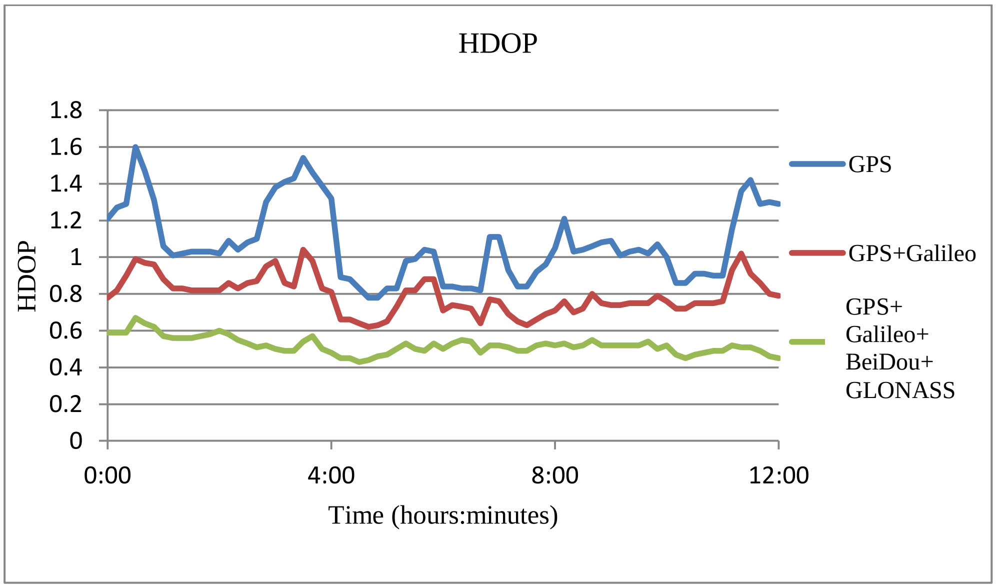

Figure 39. HDOP values over a 12-hour span for different GNSS combinations.

However, the cumulative benefits of more satellites will be smaller as more GNSS are integrated. Although testing in an urban environment for train tracking, Specht *et al*. (2020) found that while applying a two-GNSS solution considerably increased accuracy performance compared to a one-GNSS solution, the benefit of a three-GNSS solution compared to the two-GNSS solution was negligible. However, the data in Figure 39 shows that the average HDOP for GPS-only was 1.08. HDOP was reduced by 27% with the integration of Galileo, and was by 52% lower (and also more stable) upon the integration of all four GNSS. While Rychlicki, Kasprzyk & Rosinski (2020) conclude that the use of all four GNSS can improve positioning accuracy by a mere 25%, they also conclude that the integration of all four GNSS reduces convergence time by 70% compared to a single-GNSS set-up.

The selection of multi-GNSS receivers should consider what different GNSS offer. The differences in orbital configurations offer different geometry which can be advantageous in certain applications. For example, GPS+Galileo provides better vertical accuracy (altitude estimates) while the more diverse orbits of GLONASS satellites enable GPS+GLONASS receivers to provide better horizontal accuracy. If a user seeks the superior horizontal accuracy for a complex navigation system, such as that for swarm UAV operations, they can pair the receiver with a barometer or downward-facing RADAR which can help improve altitude readings.

Note that the number of satellites a receiver can track simultaneously is limited by its number of channels. In order to take advantage of all the available GNSS satellites and expedite the convergence rate accordingly, the receiver's hardware must not only be adequate in terms of numbers but also in its demodulation capabilities of difference signal types.

6.3 Other Options to Reduce Submarine TTFGF

If a GPS+Galileo receiver is impractical or unable to be installed, there are other options that range in complexity that could also help reduce a submarine's TTFGF. These options range from changes to satellite infrastructure, signal structure, and receiver capabilities.

6.3.1 Assist Data Broadcast via Satellites

A-GPS data is abundantly available on land due to the high concentration of cellular network and their wide area coverage. However, this land-based approach to expediting fixing times is impractical at sea since it depends on ground-based infrastructure. To extend coverage beyond the coasts, satellite systems in LEO like Iridium are being used to provide resilient and secure non-GNSS real-time augmentation signals through services called Satellite Time and Location (Lawrence *et al*., 2017). While their services focus on PNT solutions for those outside GNSS coverage or experiencing interference, they can also provide validation of GNSS to mitigate risk of spoofing. LEO satellites' closer proximity to the user (approximately 25 times closer than MEO) means their satellite signals can be 300 to 2400 times stronger than those from core-GNSS satellites in MEO (Lawrence *et al*., 2017). The broadcast of their own PNT signals or GNSS ephemeris and clock data would reduce the TTFF for devices specifically missing only system time as they would not have to wait through the entire navigation data message just for system time (Samson, 2011). Samson (2011) also discusses the combination of the two above strategies, with LEO satellites broadcasting system time and a special form of reduced navigation data (abbreviated almanac and ephemeris), which would reduce the TTFF in cold and warm start conditions. Lastly, the exploitation of LEO satellites would also provide more accurate Doppler positioning and velocity determination calculations due to the higher satellite rates of change (Lawrence *et al*., 2017). Finally,

In the end, a user initializing in remote regions including the middle of the ocean could download the required navigation data from these other constellations quicker than they can from the slow direct GPS satellite link. However, pairing satellite infrastructure with A-GPS broadcasts would be redundant for the majority of GPS users since they are already within A-GPS coverage. Thus the cost of this LEO endeavour would only aid those limited number of users initializing their receivers in remote regions, like submarines.

6.3.2 Satellite-station Differential System

A method of making differential correction data more accessible is through their broadcast by either GEO or LEO satellites. While differential technology would help improve real-time positioning accuracy (down to decimeter-level) and reduce fixing times for submarines in remote maritime environments, this data would likely be part of a paid subscription service (Yan, 2023). Wide-area differential data is available through LEO services such as Trimble RTX, OmniStar, StarFire, StarFix, and Atlas which can achieve positioning accuracy of 0.5 meter which benefits marine research vessels, although their subscriptions comes at a high cost (Yan, 2023). Furthermore, it would require the submarine be outfitted with a specialized receiver and an additional differential antenna (Yan, 2023).

Dedicated GEO satellites broadcasting error and differential data could alleviate the issue of the current SBAS satellites not broadcasting over remote maritime environments. The BeiDou constellation includes six satellites in geosynchronous orbit that broadcast clock error and orbit correction information that enable centimeter-level performance in offshore testing, with even better results when integrated with GPS (Yan, 2023). Expanding coverage SBAS satellites would also benefit trans-oceanic aircraft and ships, particularly as those industries transition towards autonomous vehicle control.

However, the improvement of GNSS convergence with additional LEO or GEO satellites is inherently expensive. A more cost-effective solution to improve positioning performance is the development and implementation of receiver software solutions (Rychlicki, Kasprzyk & Rosinski, 2020).

6.3.3 PPP

Research in PPP is constantly exploring methods to reduce convergence times so that centimeter-level positioning accuracy is more rapidly available; however, research is often tied to RTK capabilities to maximize its applicability (Yan & Zhang, 2022). With their lower orbit and thus faster changes in satellite geometry, Xu *et al*. (2024) examines the use of LEO satellites to augment PPP. Not only is positioning accuracy improved, but so is convergence time while TTFF is reduced (Xu *et al*., 2024). The lower latency of LEO satellites improves positioning solution quality for users seeking real-time PNT data (Hadas, Kazmierski & Sosnica, 2019).

Still, since the submarine demands accuracy and timeliness at the same time, a submarine would require real-time PPP rather than post-processing PPP which is less accurate (Yan, 2023). Currently, real-time services are still limited by predicted orbit data and require the user have a continuous datalink to access correction information (Yan, 2023). Studies like He, Cai, & Pan (2023) are exploring the use of LEO satellites to reduce PPP convergence times as a result of higher satellite geometry variation rates from lower orbits, which enhances Doppler positioning. Although simulations showed the potential to reduce convergence times to less than a minute, like differential data broadcast from GEO and LEO, this method of PPP augmentation would also likely come with paid subscription and require a separate antenna to be fitted on the submarine. There would also need to be a guarantee on the correction data quality if it were adopted for military applications.

While there are user benefits in augmenting PNT timing and accuracy with LEO satellites as well as lower launch costs, the big problem with LEO satellites is their limited coverage – approximately 1/9th the footprint area of MEO satellites (Lawrence *et al*., 2017). More satellites are required as part of a constellation to provide continuous worldwide coverage. Where a constellation of ten satellites in MEO can ensure at least one satellite is in view at all times anywhere on Earth, approximately 100 would be needed for the same conditions in LEO (Lawrence *et al*., 2017). So while LEO satellites are less expensive to launch, more of them are needed.

6.3.4 Ephemeris Models

With the orbital trajectories of GNSS satellites relatively predictable, their motion can be coupled with historical trends of clock data to model rough predictions of ephemeris data.

Fu & Lv (2021) examined the possibility of using long-term ephemeris extrapolation and clock error prediction models to reduce TTFF from minutes down to seconds. Although the research proved the method could significantly reduce convergence time, the accuracy of those fixes would be subject to the quality of the model and the time since the satellite data was collected and saved for the model. The research team also considered how to make the models available to the receiver without the submarine requiring additional hardware. Although a separate antenna would not be needed like in the cases of satellite-based A-GPS, satellite-station differential systems, or PPP, these models would need to be embedded in the receiver and thus require dedicated memory and processing.

Similarly, the concept of Self-Ephemeris uses historical ephemeris data to predict current orbital information, eliminating a receiver's need to download ephemeris data from the satellites. With the data readily available for up to 72 hours, the time for accurate positioning information can be reduced by up to 90% to an average of 3.5 seconds, mirroring the time required for a hot start (Furuno, 2014). The receiver automatically keeps the ephemeris-prediction algorithms updated with the current satellite data in the event connectivity is lost.

One problem is any positioning discrepancies when the positioning data changes over from predicted data to real data once it is available. While simulations showed that positioning errors would be small between the predicted data and real-world data, any inconsistencies would be passed onto the submarine's INS and its end-users, which may not be worth the time saved.

6.3.4 Floating GPS Antenna

With wireless signals not able to penetrate the ocean's surface, submerged submarines cannot easily receive GPS signals. One method to maintain continuous GPS connectivity could be via a floating antenna, where data is streamed to the submarine below the surface through a wire. The submarine's GPS capabilities would then mirror that of a surface ship, where positioning data is converged and works with the integrated INS for maximum positioning performance in case the submarine finds itself in either a GNSS-degraded or –denied environment.

However, a floating antenna's low profile and small size intended to for counter-detection purposes may subject the antenna to signal attenuation. The GPS receiver would experience outages whenever the antenna would be awash, just like when the submarine's fitted antenna submerges. While submarines can be equipped with antenna buoys to receive VLF communications (3-30 kHz) while underwater, VLF communications are not attenuated by water as much as GPS signals; instead they able to be recovered at depths of a few meters.

Furthermore, the position estimate from a floating antenna would not accurately represent the submarine's position underwater as a result from any slack in the wire (Figure 40). Tracking and monitoring techniques could be employed, although at the cost of additional complexity. If not, the submarine could still accurately navigate underwater via the SINS while the antenna maintains connectivity, keeping the receiver in the hot state like a surface ship or worse case, the warm state when the antenna is occasionally awash for short periods of time.

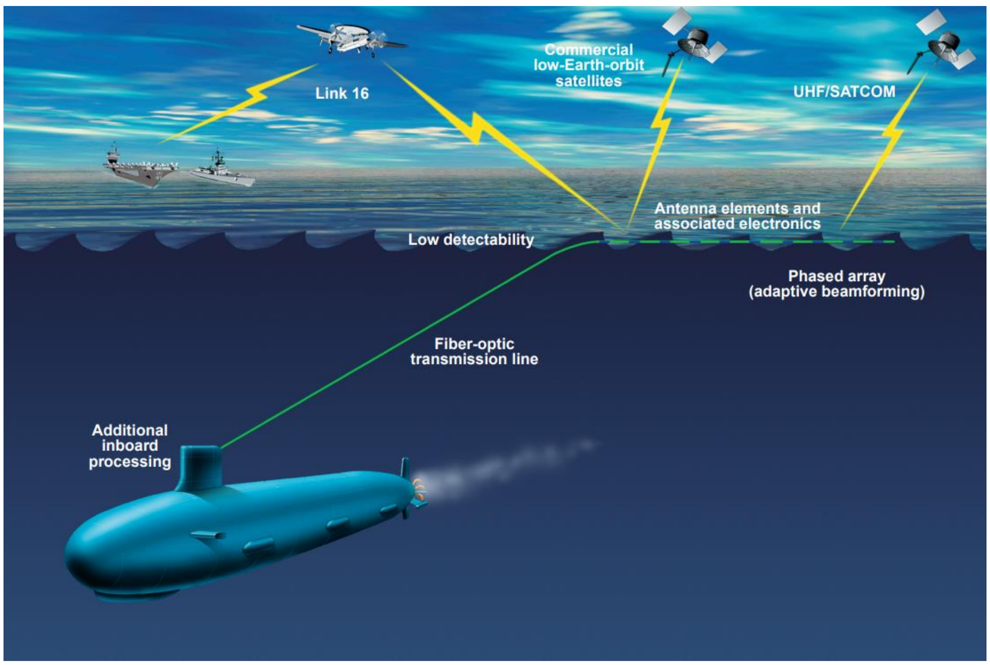

Figure 40. Floating antennas streamed to the surface can help submarines maintain connectivity, including GPS connectivity (Thompson *et al*., 1999).

6.3.6 GPS Data via Acoustic Signals

Another method to maintain GPS connectivity is via surface infrastructure such as ocean buoys or even ships that span the air-water boundary. Infrastructure or vehicles on the water and with GPS connectivity could receive the wireless GPS signals and translate them into acoustic signals, where they could then be received by submerged submarines. If accurate positioning is not able to be maintained by the submarine, it could still keep the ephemeris and clock data up to date, reducing the fixing time whenever it returns to the surface.

Ships and submarines regularly sail together as part of naval task groups, giving the submarine a friendly source for GPS data even in remote maritime environments. However, the translation of GPS data into acoustic signals would require additional processes not currently employed. With that capability, a ship's continuous broadcast would advertise a friendly submarine's presence in the area, as well as advertise its own position via a larger acoustic signature.

Submerged position updates can also be achieved through underwater transponder positioning (UTP). Working with a surface ship with GPS positioning, a submerged vessel can estimate its position with the ship's GPS data and its relative acoustic positioning to the mother ship via underwater transponders (Jalving, 2005). This acoustic range and bearing positioning method is used by autonomous underwater vehicles, but is limited by the acoustic range of the surface ship.

Alternatively, ships could share GPS data over voice channels, acting as A-GPS infrastructure when the submarine is either at PD or surfaced, expediting its fixing time and convergence. The submarine would then experience the same TTFF and convergence as receivers on land near A-GPS infrastructure. Similar faster broadcasts over A-GPS channels that have cut mobile devices' TTFF from minutes down to a matter of seconds (Shokouh, 2013; Zekavat & Buehrer, 2012) would greatly benefit submarine navigation.

6.4 The Future and Recommendations

This section discusses possible avenues for future GNSS studies in the maritime environment and recommendations to ameliorate submarine navigation.

6.4.1 The Future of GNSS

Research into improving GNSS and inertial navigation performance are continuously moving forward as both systems play critical roles in complex and integrated navigation system. While integrated GPS/INS have navigated ships, submarines, and aircraft (both crewed and uncrewed) for years, a relatively new role they will play is that in autonomous car (Loizou, 2020).

Current GNSS receiver performance, including the convergence rates seen in this research, are only expected to improve as new satellites are developed and launched. The U.S. DoD has publically committed to maintain GPS service so as to retain the leadership of space-based PNT services while also remaining open to cooperation for GNSS interoperability and augmentation (National Coordination Office for Space-Based Positioning, Navigation, and Timing, 2022). GPS modernization should deliver new satellites with more diverse signals, like those available with Galileo, which could better support the user's PNT needs. One example of signal improvement is the new GPS M-code. In 2023, Canada was the first US-ally to receive and test an M-code enabled receiver (Government of Canada, 2023b). In addition to the improved anti-jamming and anti-spoofing capabilities of M-code, these receivers should experience more accurate and quicker GPS performance.

As for Galileo, which is still in its initial operational capability phase, new satellites are being launched on a regular basis; as newer satellites are added to the constellation, performance is expected to ameliorate (FAA, 2021). Initial in-orbit test reviews of new satellites in 2021 revealed increases in positioning accuracy and system robustness of the entire constellation (Cozzens, 2022a). As upgrades are incorporated into every satellite variant, they will slowly update the constellation with every launch. Although changes to new satellites must be compatible with existing satellites still in operation, their associated ground segment, and also in-market chipsets (Cozzens, 2022a). Satellite signals should also be optimized for demodulation, convergence, accuracy, and interoperability, thus maximizing their application by users.

In 2023, the European Union Agency for the Space Programme was set to upgrade the Galileo OS further with improved clock and ephemeris data, synchronization, and error correction (European GNSS Agency, 2021). These improvements to the system, particularly to time synchronization, were set to enable a faster TTFF (European GNSS Agency, 2021; Hubert, 2022). With the Galileo specifically designed and continuously seeking low fixing times, in that not all the information is required by the receiver immediately, the GPS data message could similarly be re-designed. However, any changes to GNSS operability as they are upgraded will have to be backwards compatible with existing receivers (EUSPA, 2022a).

Within the user segment, GNSS applications require common reference frames to comply with different systems' navigation solutions, thus ensuring they can take advantage of the abundance of available GNSS and signals of opportunity. In order to remain relevant and maximize commercial applications, multi-GNSS receivers should be kept low cost while GNSS signals should be designed to be both compatible[31] and interoperable[32].

This thesis replicated the submarine's conditions as much as possible; the results could be confirmed by follow-on research onboard a submarine at sea, although the operational nature of submarine would make this endeavour a challenge for academia. While using more satellites gives increased satellite availability and better DOP which expedite convergence, novel "satellite intelligence" algorithms could help receivers use the most optimally-positioned satellites for positioning, depending on the user's priority (two-dimensional

[31] One GNSS shouldn't degrade or compromise another.
[32] Use of common center frequencies and use of similar demodulation requirements.

positioning, three-dimensional positioning, velocity only, etc.). Current multi-GNSS receivers do not prioritize a specific constellation or particular satellites over others; however, prioritization could help further improve positioning accuracy. Satellite closest to the tetrahedron shape for low DOP, with one satellite at the zenith and three satellites equally-spaced above the horizon, could be prioritized for optimal three-dimensional positioning. Likewise for a user seeking optimal vertical or horizontal positioning, a receiver could seek the satellites that minimize the DOP for that objective. Studies could also examine improvements to orbital configurations. As Earth's climate changes and humanity inhabits more of the Polar Regions, the orbital configuration of the existing GNSS could also be re-organized as to optimize satellite availability and DOP for all regions. Studies could also examine the optimization of satellite coverage and GNSS combinations, not just on Earth but also other planetary systems as we strive for habitation on other surfaces.

6.4.2 The Future of Submarine Navigation

Particularly for submarine positioning, advances in receiver accuracy and inertial navigation performance will reduce the number of times a submarine gets forced to the surface for an accurate position update in the first place. Highly-accurate accelerometers measuring gravity gradients, as well as the submarine's dynamics on all axes as it's subjected to currents will reduce the POE growth (Schmidt, 2015).

Revolutions in quantum, quartz silicon micro-mechanical, and fiber-optic sensors as well as MEMS devices, and cold atom interferometry could make future INS magnitudes more accurate (Schmidt, 2015). These systems and others could extend the period an INS can run without GPS updates, minimizing the numbers of times a submarine is made vulnerable at or near the surface. However, a smaller reliance on a GNSS receiver for position updates as INS technology improves and is more costly means that the cost performance of the GNSS receiver compared to the INS becomes smaller to the point is nears being relatively insignificant (Schmidt, 2015). A comparison between inertial technology, their performance, and cost is presented in Figure 41.

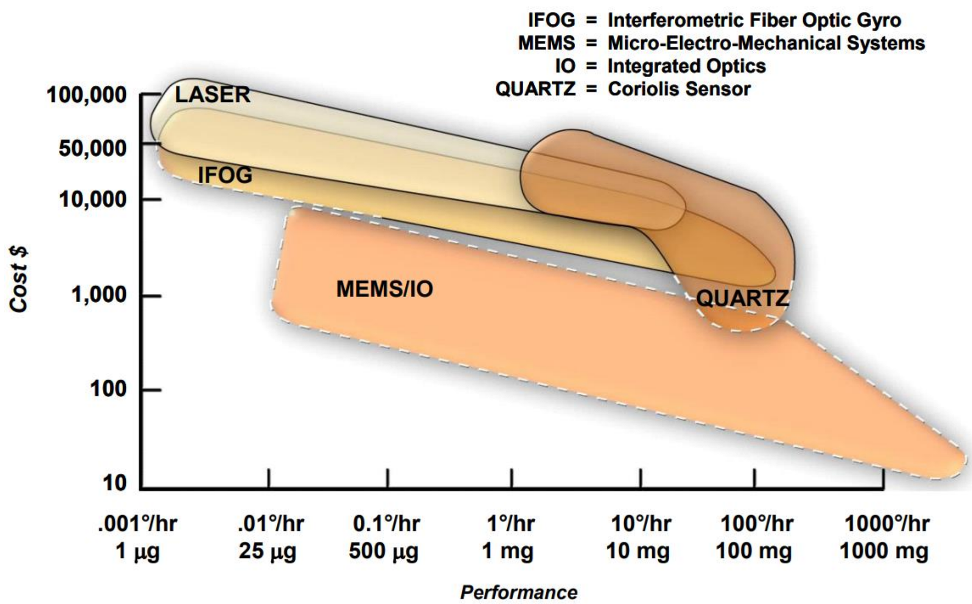

Figure 41. Price range of integrated GPS/INS technologies with their associated long-term accuracy (Schmidt, 2015).

Researchers are also seeking novel positioning techniques for underwater vehicles – both crewed and uncrewed – which could also be applied to underwater surveying, drilling, and research. Jalving (2005) discussed terrain mapping/bottom-contour navigation via multibeam echo sounders to refine positioning while underwater and Rogobete (2018) showed the promising results of using gravity potential fields for the same objective. With both of these techniques dependent on digital maps paired with the technology, both are hampered by the limited survey state of the world's oceans. Furthermore, the use of sonar to determine position demands the transmission of energy into the surrounding waters that can compromise a submarine's stealth. As these capabilities are refined and the seabed/gravity field more accurately mapped, they could be integrated into the submarine's GPS/INS, similar to the integration of Galileo, and help overcome positioning challenges in GPS-denied environments (Jalving, 2005).

Chapter 7

Conclusion

In order to remain covert, a submarine seeks the lowest time possible between antenna exposure and the calculation of accurate positioning data that meets the integrated GPS/INS accuracy threshold - duration termed the Time To First Good Fix (TTFGF) in this paper. Compared to a GPS-only receiver, this thesis investigated if a GPS+Galileo receiver could reduce a submarine's wait time for accurate positioning information upon returning to the surface.

It was found that for 4-meter positioning accuracy, a GPS+Galileo receiver produces accurate positioning data 27 seconds faster than a GPS-only receiver, reducing the wait time for accurate data by 45%. Despite the GPS-only receiver always outputting a position estimate quicker, the GPS+Galileo receiver was always more accurate and provided accurate positioning data quicker regardless of the user's accuracy threshold. Despite differences in the two GNSS, including data message structures and data rates which results in greater signal diversity when combined, the interoperability of the two systems results in increased satellite availability and more optimal satellite geometry which expedites solution convergence.

In addition to the quicker convergence rate, the implementation of multi-GNSS receivers on submarines is further warranted by their delivery of increased positioning accuracy, greater positioning data stability, and enhanced system redundancy for improved performance. The interoperability, coupled with the availability of multi-GNSS technology and their worldwide availability makes a GPS+Galileo receiver a forerunner in positioning-augmentation strategies, particularly for remote maritime environments. A GPS+Galileo receiver is a low-cost approach that could be applied to submarines with little change to infrastructure but high returns in submarine navigation and operational capability.

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Appendices

Appendix A – TTFF and TTFGF Values (0 km/h)

Table 11. GPS receiver with an accuracy threshold of 13 feet / 4 meters.

Test #	TTFF (min:sec)	TTFGF (min:sec)	Number of Visible Satellites	HDOP
1	0:05	0:47	9	0.92
2	0:08	0:41	10	0.88
3	0:04	1:04	8	0.97
4	0:05	1:10	10	0.88
5	0:06	0:46	8	0.98
6	0:02	0:39	9	1.01
7	0:05	0:55	8	1.04
8	0:03	0:46	9	0.92
9	0:02	1:07	11	0.79
10	0:05	0:33	12	0.76
11	0:03	0:58	10	0.85
12	0:04	1:41	8	1.02
13	0:04	1:51	8	0.97
14	0:03	1:12	8	0.97
15	0:02	1:04	9	0.93
16	0:03	1:30	10	0.86
17	0:03	0:54	9	0.94
18	0:06	0:37	9	0.91
19	0:03	1:01	10	0.86
20	0:04	0:59	9	0.96
21	0:04	1:04	8	1.01
22	0:03	1:00	8	1.03
23	0:02	0:43	11	0.77
24	0:02	0:59	12	0.74
25	0:03	0:54	10	0.85

Table 12. GPS+Galileo receiver with an accuracy threshold of 13 feet / 4 meters.

Test #	TTFF (min:sec)	TTFGF (min:sec)	Number of Visible Satellites	HDOP
1	0:08	0:35	15	0.73
2	0:08	0:19	18	0.62
3	0:05	0:37	18	0.68
4	0:05	0:28	17	0.70
5	0:08	0:23	16	0.70
6	0:05	0:24	17	0.69
7	0:07	0:31	14	0.74
8	0:06	0:16	17	0.68
9	0:06	0:15	19	0.60
10	0:06	0:14	19	0.62
11	0:06	0:27	18	0.66
12	0:07	0:51	16	0.70
13	0:07	0:41	17	0.70
14	0:06	0:41	18	0.69
15	0:06	0:30	17	0.69
16	0:06	0:35	17	0.67
17	0:07	0:41	18	0.65
18	0:05	0:31	15	0.76
19	0:07	0:33	17	0.66
20	0:07	0:32	14	0.79
21	0:07	0:24	17	0.66
22	0:06	0:33	17	0.65
23	0:05	0:34	23	0.53
24	0:06	0:17	23	0.56
25	0:08	0:17	15	0.63

Appendix B – TTFF and TTFGF Values (20 km/h)

Table 13. GPS receiver with an accuracy threshold of 13 feet / 4 meters.

Test #	TTFF (min:sec)	TTFGF (min:sec)	Number of Visible Satellites	HDOP
1	0:06	1:16	10	0.93
2	0:04	0:36	12	0.74
3	0:04	0:46	11	0.78
4	0:04	1:13	11	0.77
5	0:03	1:12	11	0.77
6	0:03	0:48	9	0.92
7	0:06	0:52	8	1.06
8	0:04	0:39	10	0.86
9	0:03	0:35	8	1.06
10	0:02	1:04	10	0.85
11	0:02	1:33	10	0.87
12	0:03	1:31	8	0.97

Table 14. GPS+Galileo receiver with an accuracy threshold of 13 feet / 4 meters.

Test #	TTFF (min:sec)	TTFGF (min:sec)	Number of Visible Satellites	HDOP
1	0:06	0:36	20	0.59
2	0:06	0:16	21	0.54
3	0:07	0:17	22	0.56
4	0:07	0:28	18	0.60
5	0:07	0:22	19	0.62
6	0:06	0:41	17	0.68
7	0:07	0:33	12	0.91
8	0:05	0:31	17	0.70
9	0:07	0:18	13	0.87
10	0:05	0:26	17	0.69
11	0:06	0:45	17	0.70
12	0:04	0:36	18	0.69